

Recent Developments in Lattice QCD

Andreas S. Kronfeld



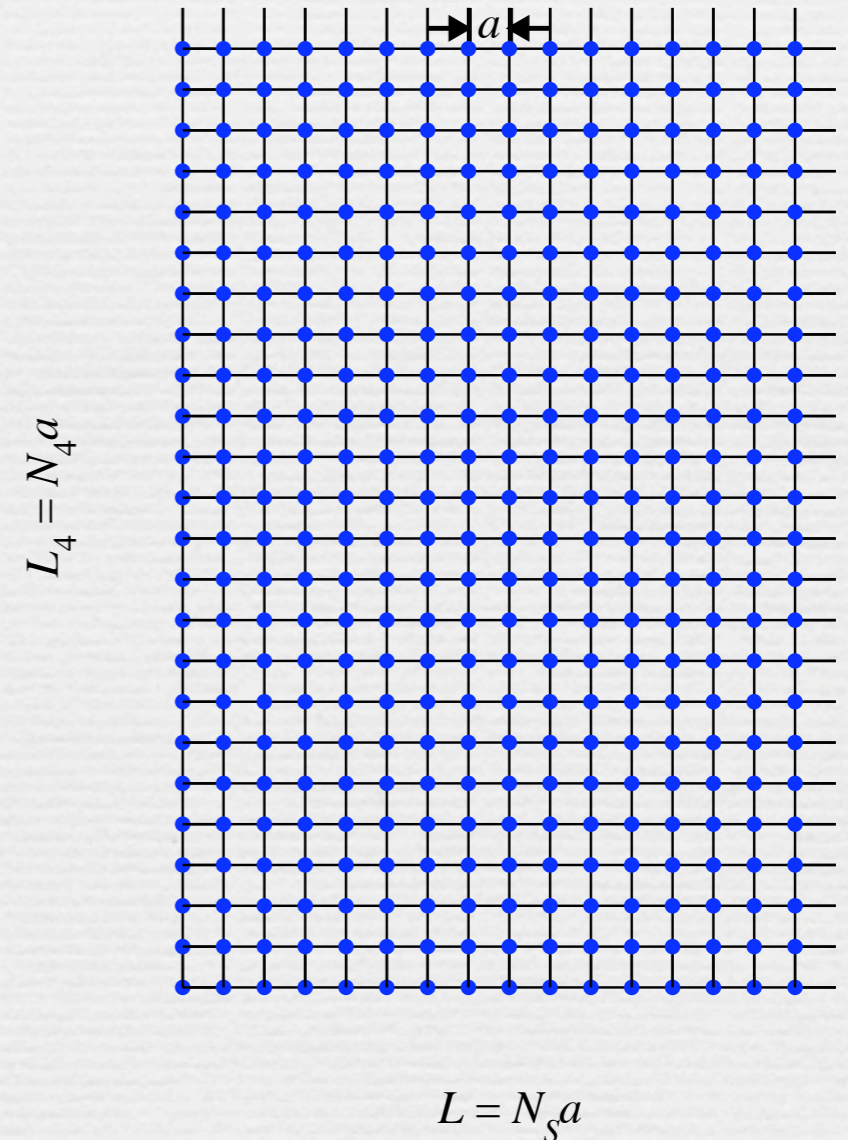
April Meeting—May 2–5, 2009

Lattice QCD

Lattice Gauge Theory

$$\langle \bullet \rangle = \frac{1}{Z} \int \mathcal{D}U \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp(-S) [\bullet]$$

- Infinite continuum: uncountably many d.o.f.
- Infinite lattice: countably many; used to define QFT
- Finite lattice: can evaluate integrals on a computer; dimension $\sim 10^8$

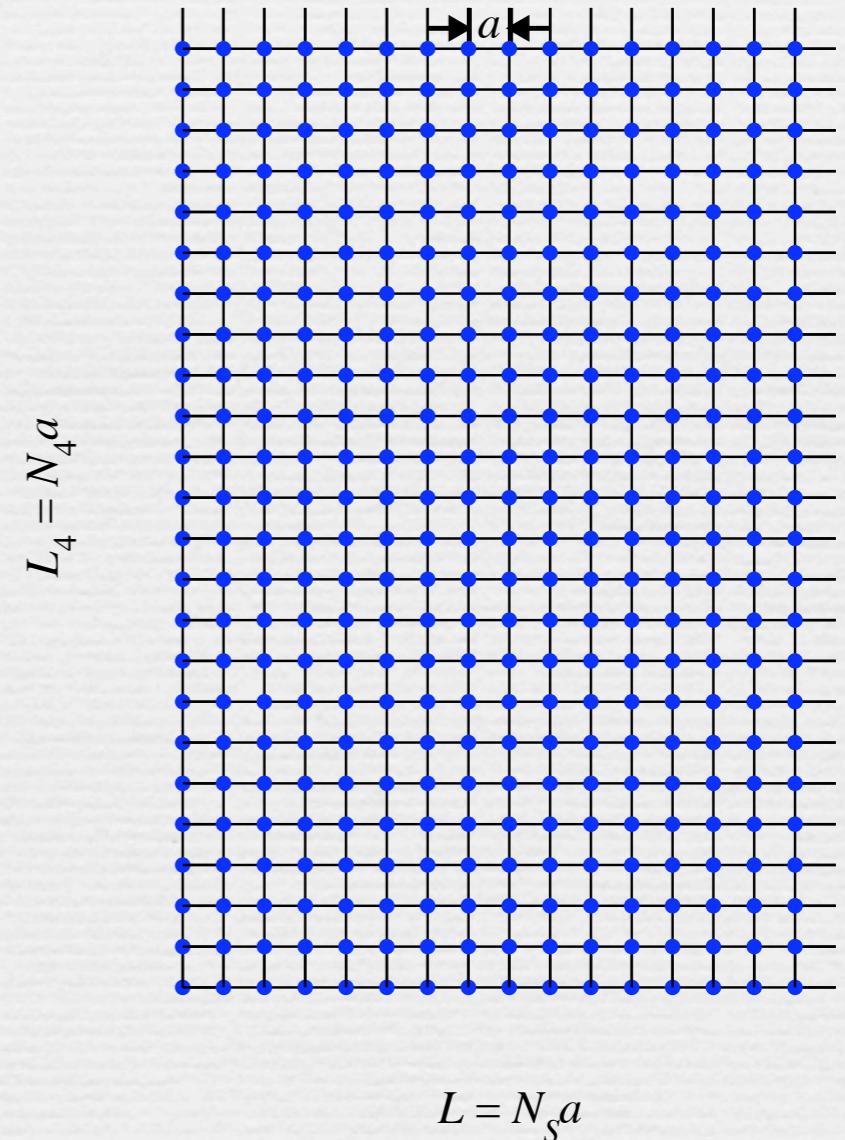


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hand

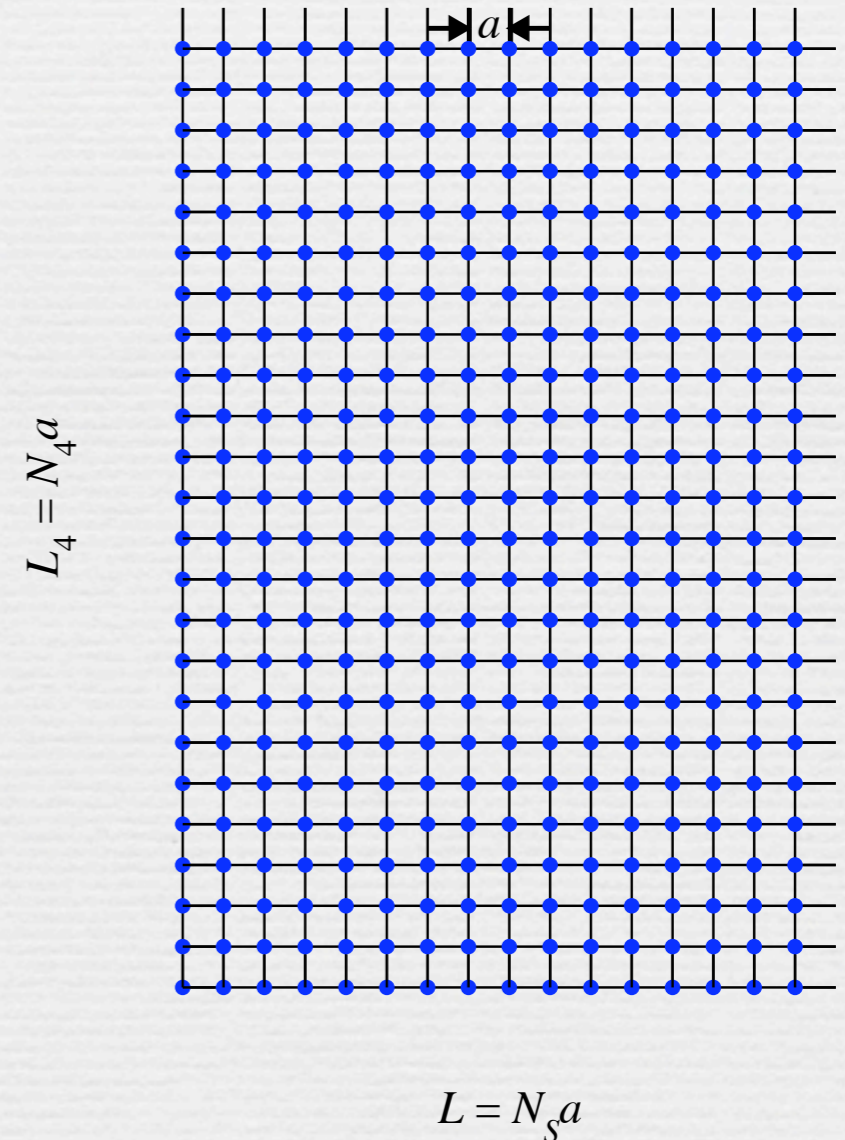
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Lattice Gauge Theory

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Some Jargon

- QCD observables (quark integrals by hand):

$$\langle \bullet \rangle = \frac{1}{Z} \int \mathcal{D}U \prod_{f=1}^{n_f} \det(\not{D} + m_f) \exp(-S_{\text{gauge}}) [\bullet]$$

- *Quenched* means replace det with 1.
- *Unquenched* means not to do that.
- *Partially* quenched doesn't mean " n_f too small" but $m_{\text{val}} \neq m_{\text{sea}}$, or even $\not{D}_{\text{val}} \neq \not{D}_{\text{sea}}$ ("mixed action").

Sea Quarks

- Staggered quarks, with rooted determinant, $O(a^2)$.
- Wilson quarks, $O(a)$:
 - tree or nonperturbatively $O(a)$ improved;
 - twisted mass term — auto $O(a)$ improvement.
- Ginsparg-Wilson (domain wall or overlap), $O(a^2)$:
 - $D\gamma_5 + \gamma_5 D = 2aD^2$ implemented w/ $\text{sign}(D_W)$.

- ✧ Many numerical simulations with sea quarks are called (perhaps misleadingly) “full QCD.”
 - ✧ $n_f = 2$: with *same* mass, omitting strange sea;
 - ✧ $n_f = 3$: may (or may not) imply 3 of *same* mass;
 - ✧ $n_f = 2+1$: strange sea + two as light as possible;
 - ✧ $n_f = 2+1+1$: add charmed sea to 2+1.

- ✧ “Full QCD” can also mean $m_{\text{val}} = m_{\text{sea}}$, $D_{\text{val}} = D_{\text{sea}}$.

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Correlators

- Two-point functions for masses $\pi(t) = \bar{\Psi}_u \gamma_5 S \Psi_d$:

$$\langle \pi(t) \pi^\dagger(0) \rangle = \sum_n |\langle 0 | \hat{\pi} | \pi_n \rangle|^2 \exp(-m_{\pi_n} t)$$

- Two-point functions for decay constants:

$$\langle J(t) \pi^\dagger(0) \rangle = \sum_n \langle 0 | \hat{J} | \pi_n \rangle \langle \pi_n | \hat{\pi}^\dagger | 0 \rangle \exp(-m_{\pi_n} t)$$

- Three-point functions for form factors, mixing:

$$\begin{aligned} \langle \pi(t) J(u) B^\dagger(0) \rangle &= \sum_{mn} \langle 0 | \hat{\pi} | \pi_m \rangle \langle \pi_n | \hat{J} | B_m \rangle \langle B_m | \hat{B}^\dagger | 0 \rangle \\ &\quad \times \exp[-m_{\pi_n}(t-u) - m_{B_m} u] \end{aligned}$$

Scope of this talk

- Lattice gauge theory is now a broad field:
 - SM parameters and flavor physics;
 - nucleon properties and excited baryons;
 - hadron-hadron interactions;
 - QCD thermodynamics;
 - walking QCD, super YM, Higgs, BSM.
- USQCD overview, arXiv:0807.2220.

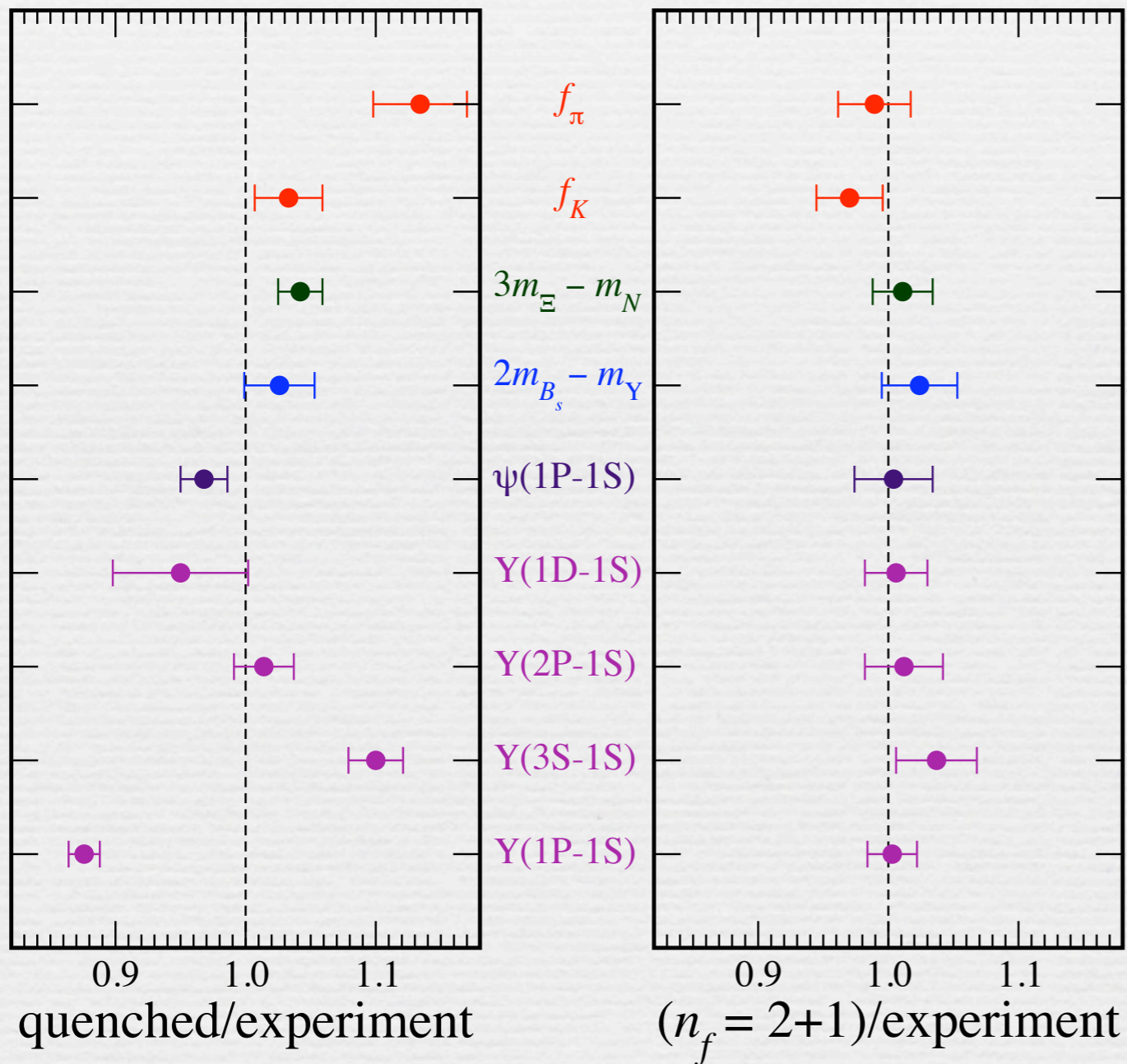
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Hadron Spectrum

2+1 Sea Quarks!

HPQCD, MILC, Fermilab Lattice, hep-lat/0304004



• $a = 0.12$ & 0.09 fm

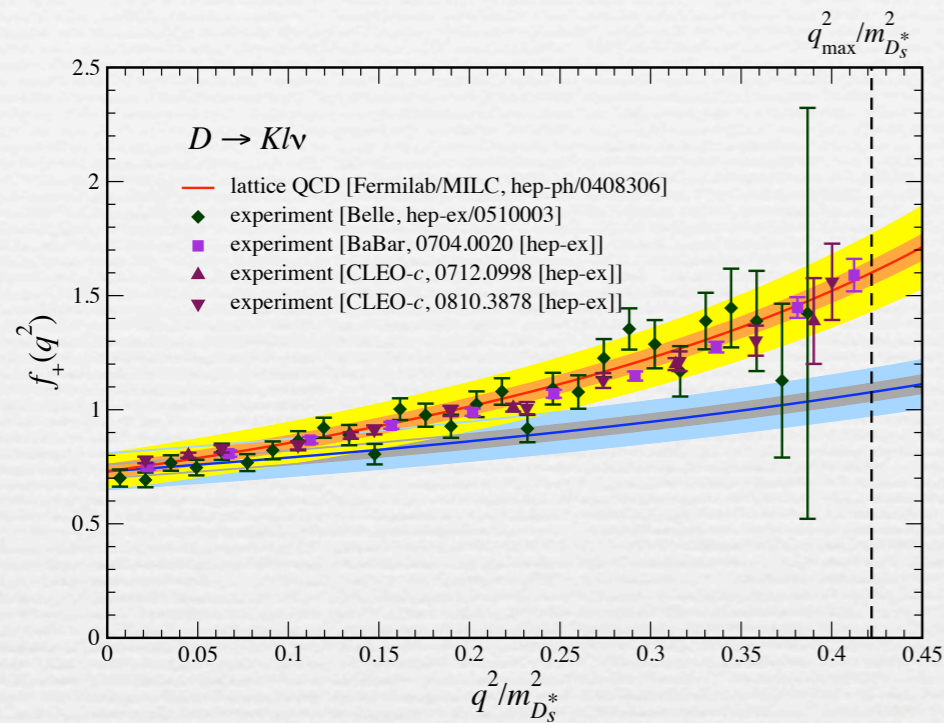
• $O(a^2)$ improved

• FAT7 smearing

• $2m_l < m_q < m_s$

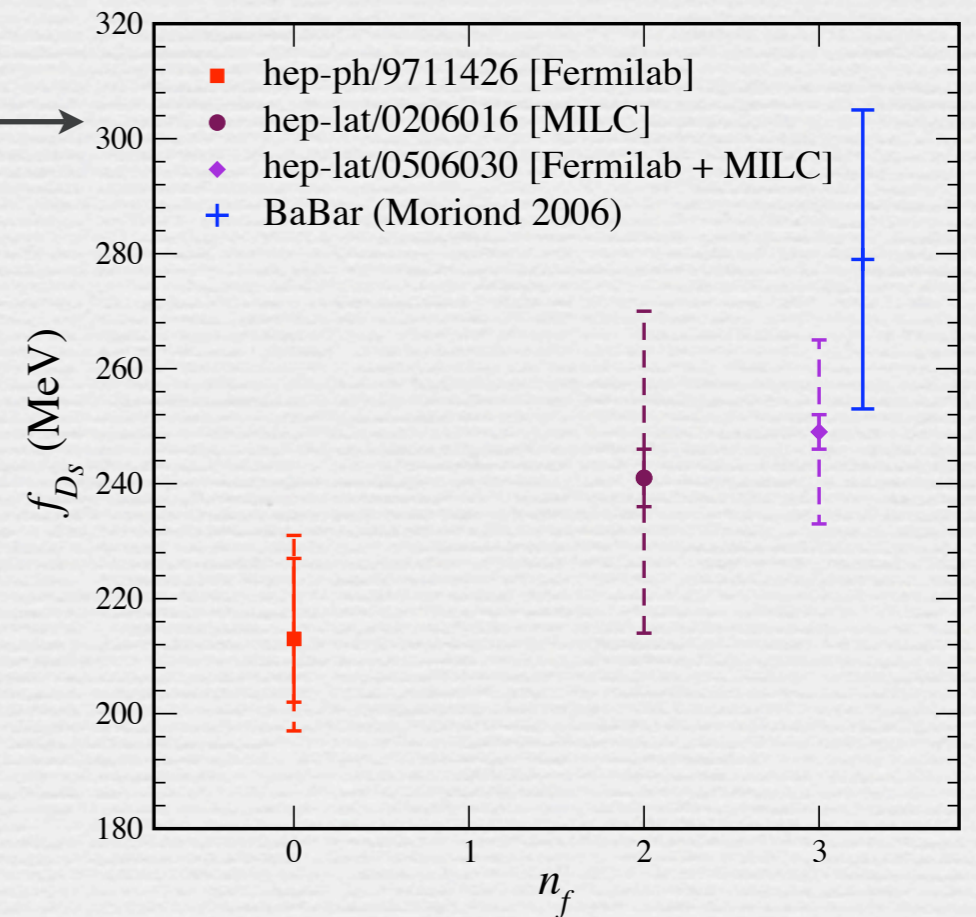
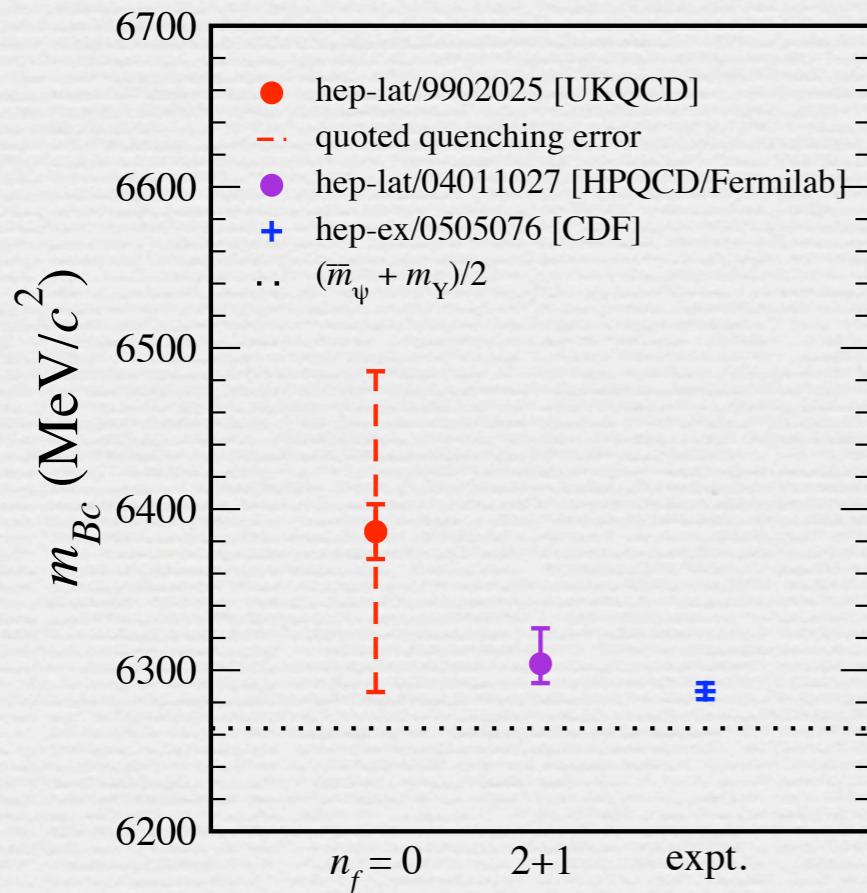
• $\pi, K, Y(2S)$ input

Predictions



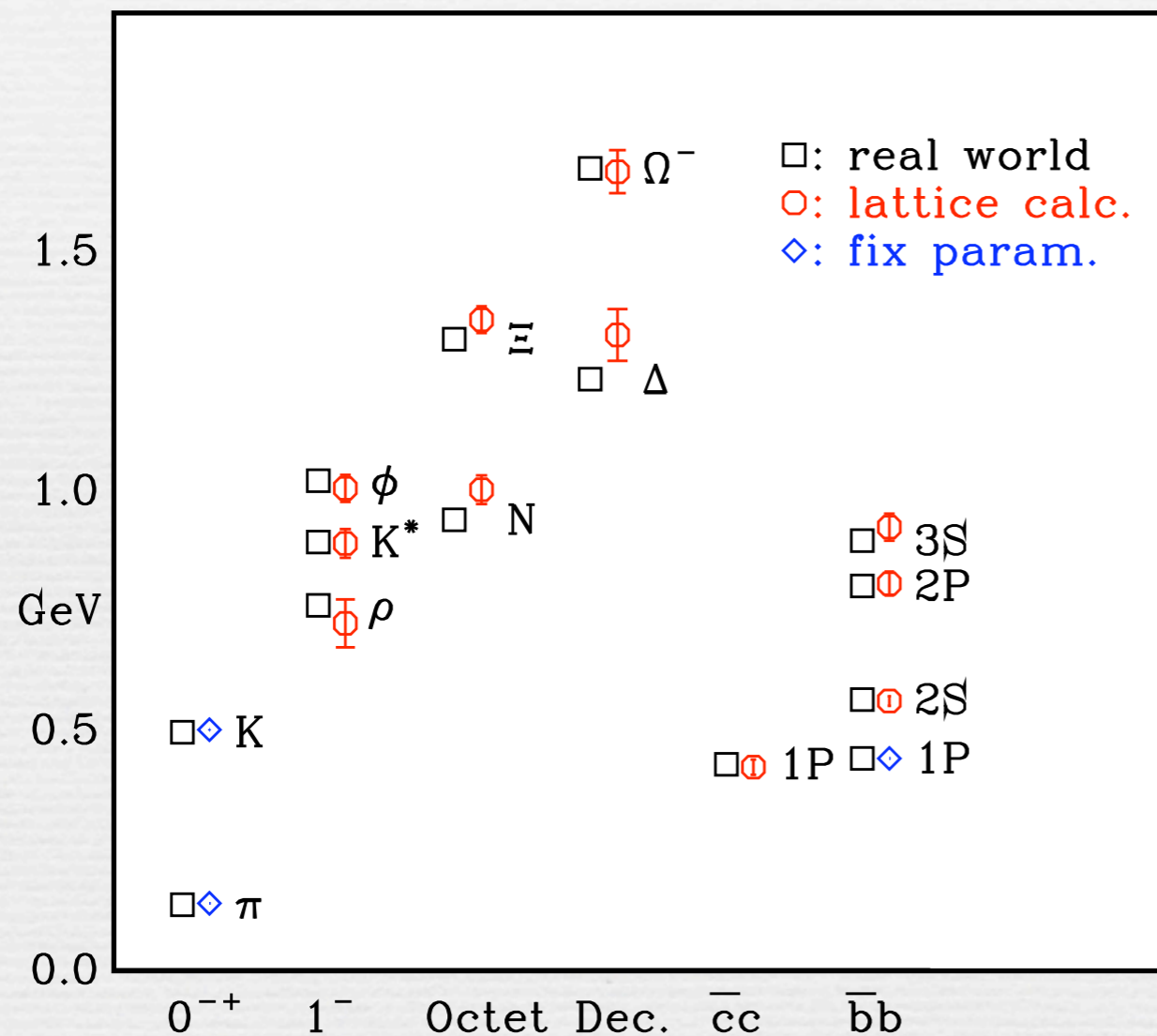
- ☞ Semileptonic form factor for $D \rightarrow Klv$
- ☞ Mass of B_c meson
- ☞ Charmed decay constants

2004
2005



Hadron Spectrum 1

MILC Col'n, *PRD* 70, 094505 (2004); arXiv:0903.3598



☞ $a = 0.12$ & 0.09 fm

☞ $O(a^2)$ staggered

☞ FAT7 smearing

☞ $2m_l < m_q < m_s$

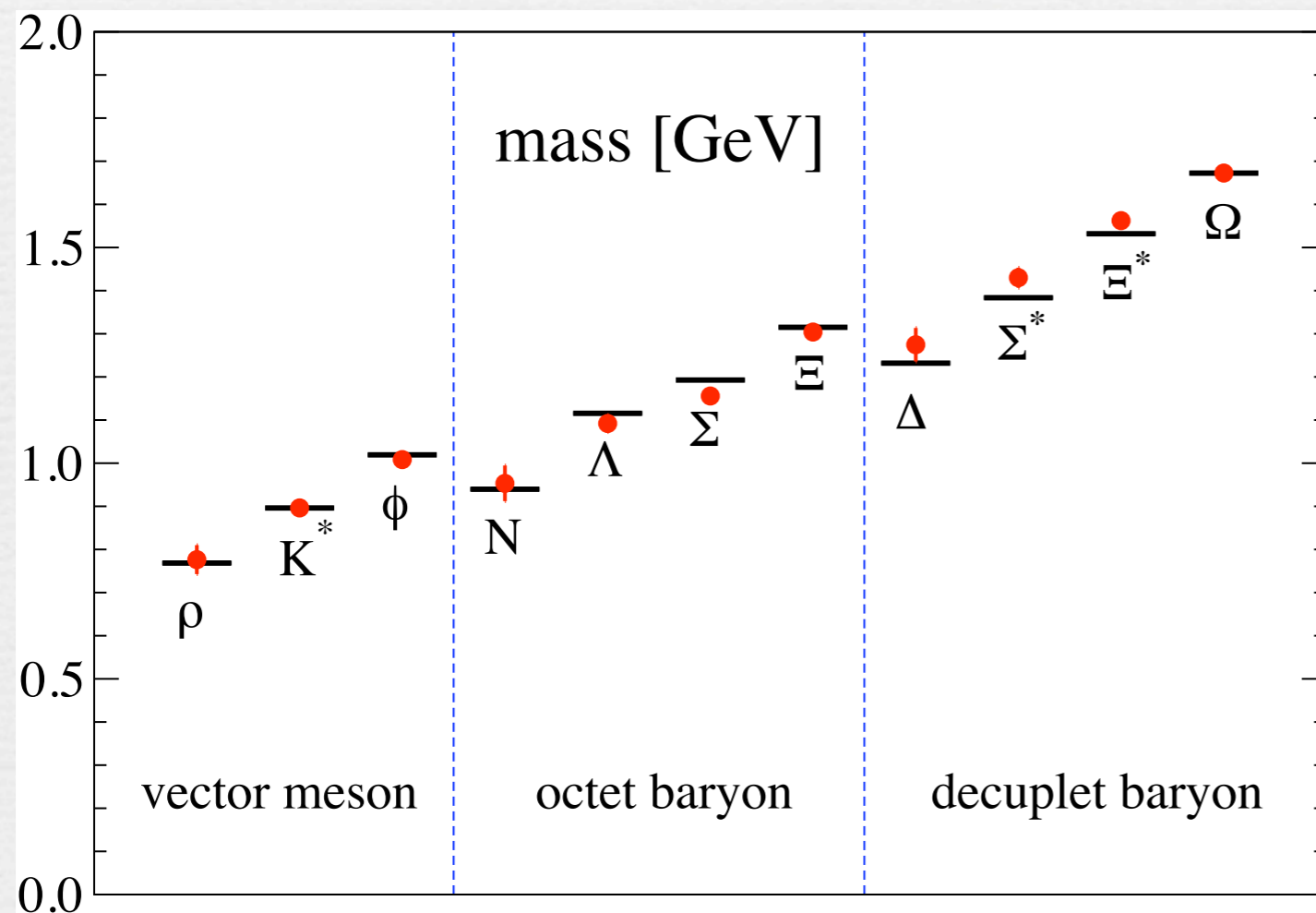
☞ $\pi, K, Y(1P)$ input

QCD postdicts the low-lying hadron masses!

Hadron Spectrum 2

PACS-CS Collaboration, *PRD* 79, 034503 (2009).

cf. earlier work by CP-PACS



$a = 0.091$ fm

NP $O(a)$ Wilson

no smearing

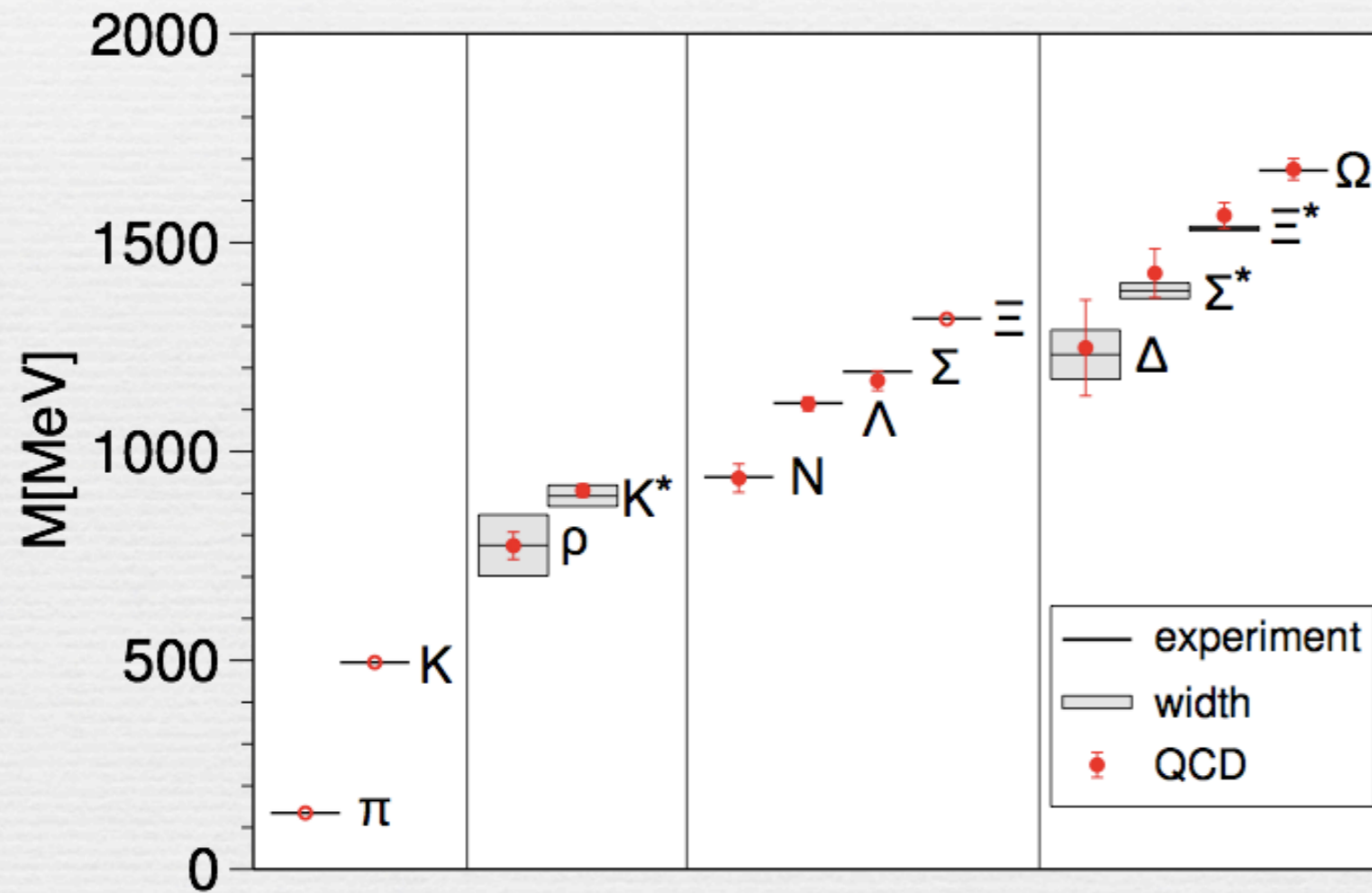
$m_q \approx 1.3m_l$

π, K, Ω input

QCD postdicts the low-lying hadron masses!

Hadron Spectrum 3

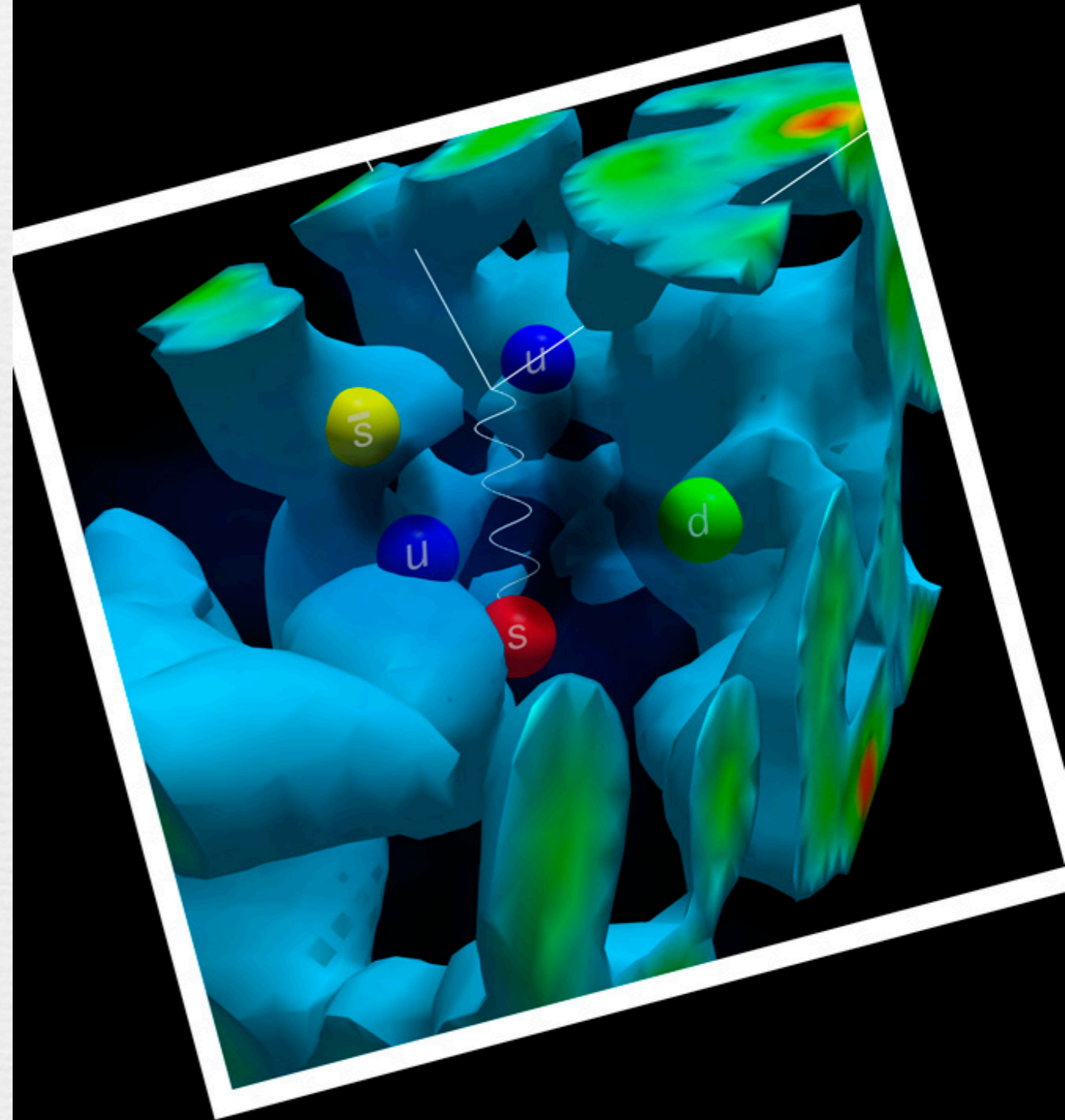
BMW Collaboration: *Science* 322, 1224 (2008).



- $a = 0.125, 0.085,$
& 0.065 fm
- tree $O(a)$ Wilson
- $6\times$ stout smearing
- $2m_l < m_q < 1.7m_s$
- π, K, Ξ input

QCD postdicts the low-lying hadron masses!

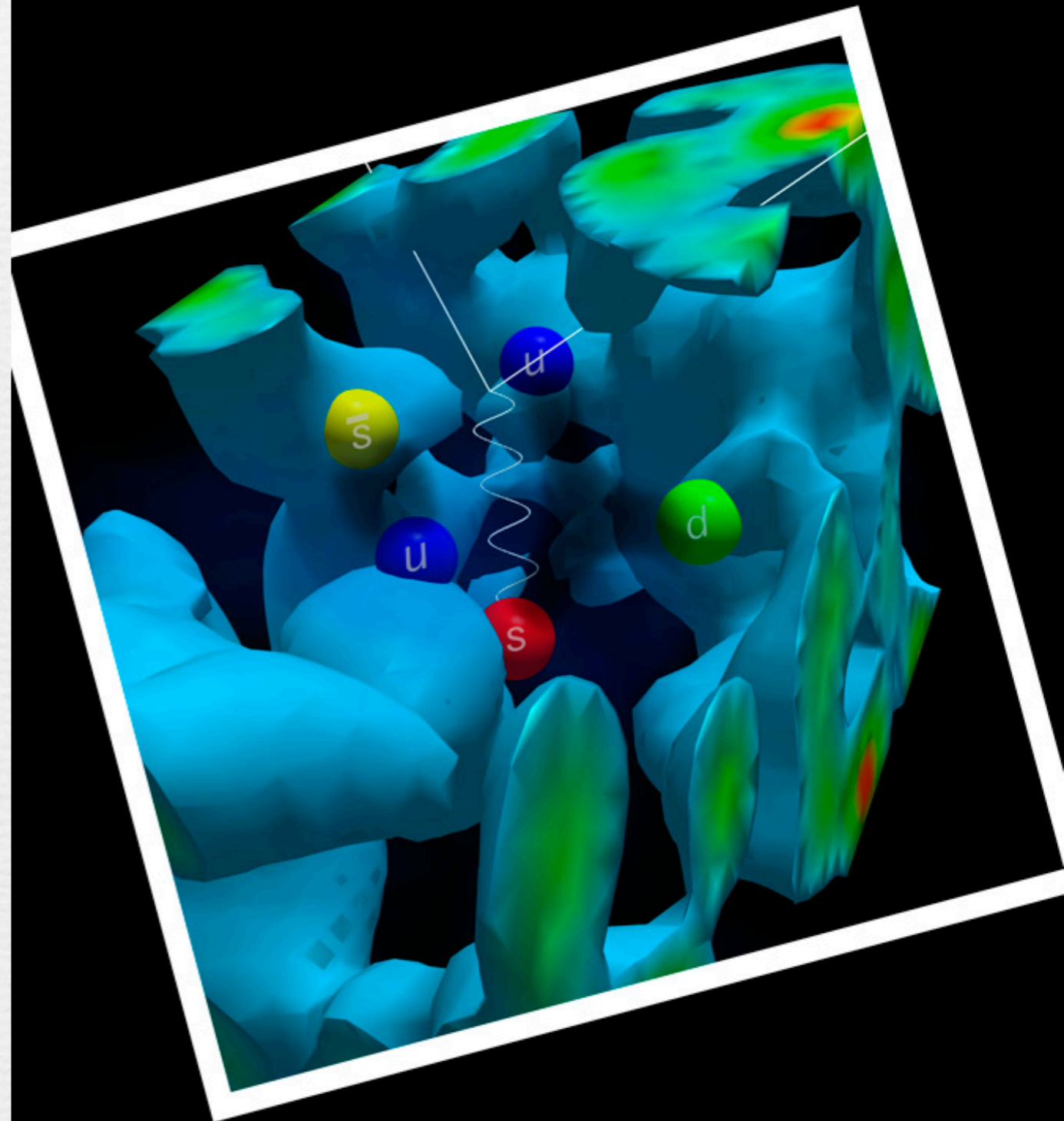
$$m = E/c^2$$



$$m = E/c^2$$



The source of your weight problem is quantum chromodynamics



QCD Parameters

Quark Masses & α_s

- Light quark masses (MILC+HPQCD):

$$m_u = 1.9 \pm 0.2 \text{ MeV},$$

$$m_d = 4.6 \pm 0.3 \text{ MeV},$$

$$m_s = 88 \pm 5 \text{ MeV}.$$

with two-loop matching.

- Heavy quark masses (next slides).
- Strong coupling α_s (after that).

Charmed Quark Mass

HPQCD+Karlsruhe, arXiv:0805.2999

- Moments of charmonium correlators:

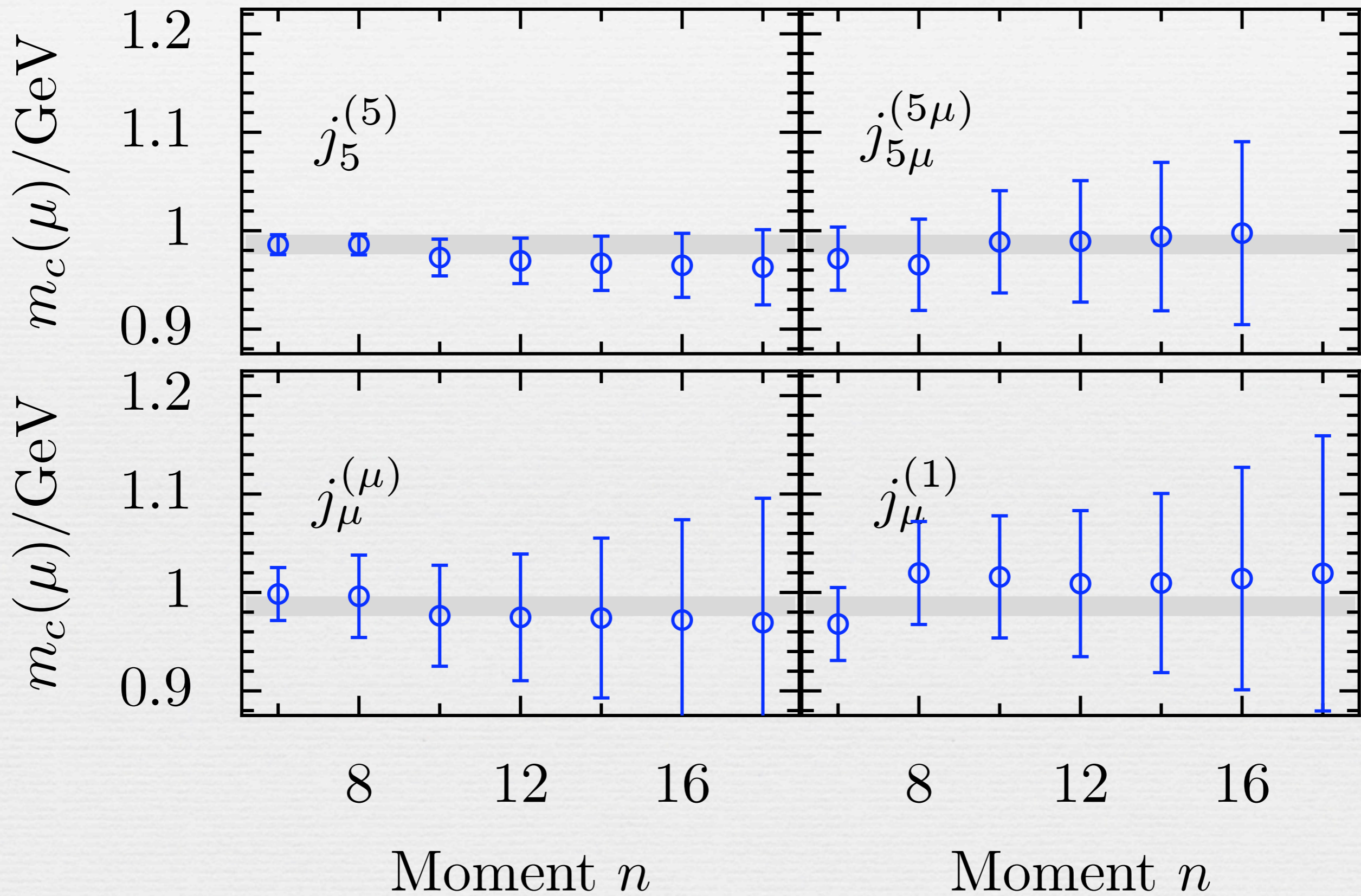
$$G_n = \sum_{\mathbf{x}, t} t^n \langle m_c \bar{c} \gamma_5 c(\mathbf{x}, t) m_c \bar{c} \gamma_5 c(\mathbf{0}, 0) \rangle$$

computed with lattice QCD *and* with continuum perturbation theory yield quark mass and α_s
[Bochkarev & de Forcrand, hep-lat/9505025].

- Any channel would do; like using measurements of $e^+e^- \rightarrow$ hadrons for vector-vector channel.

- ✧ Several tactics to reduce discretization effects to $O(\alpha_s(m_c a)^{2,4,\dots})$.
- ✧ HPQCD: HISQ valence on 2+1 asqtad sea, with $a = 0.15, 0.12, 0.09, 0.06$ fm.
- ✧ Karlsruhe: PT for moments through α_s^3 .
- ✧ From G_6 & G_8 of $j_5 j_5$: $m_c(m_c) = 1.268(9)$ GeV.
- ✧ Compare $e^+e^- j_{\mu} j_{\mu}$: $m_c(m_c) = 1.268(12)$ GeV.

$\mu = 3 \text{ GeV}$



Strong Coupling α_s

Charmonium moments:

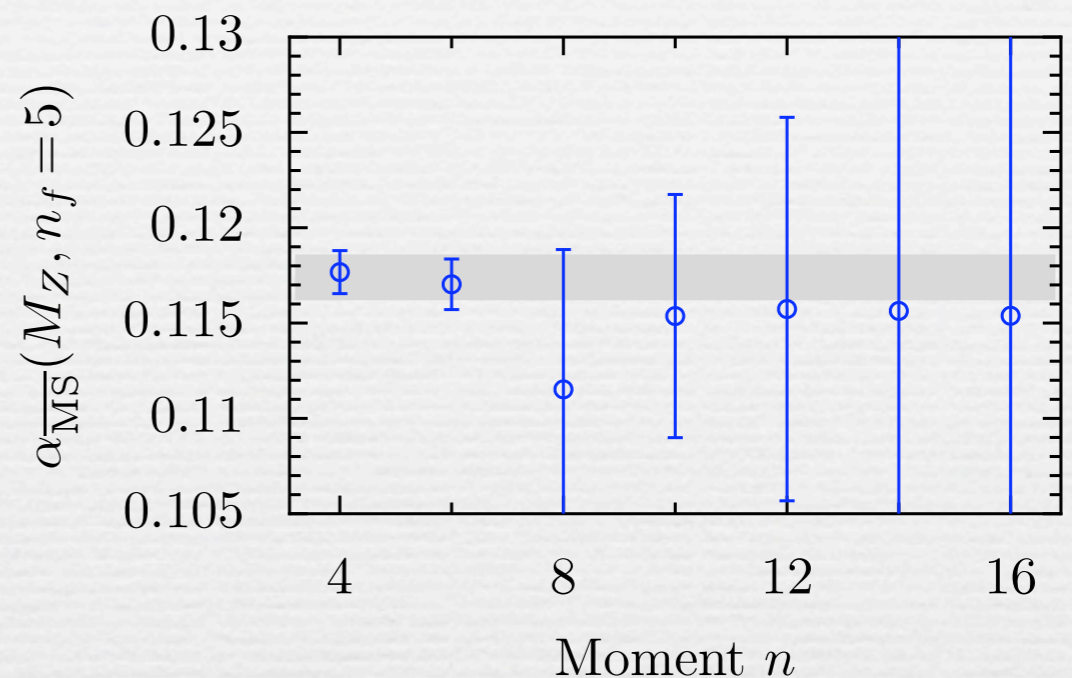
$\alpha_s = 0.1174(12)$

Wilson loops:

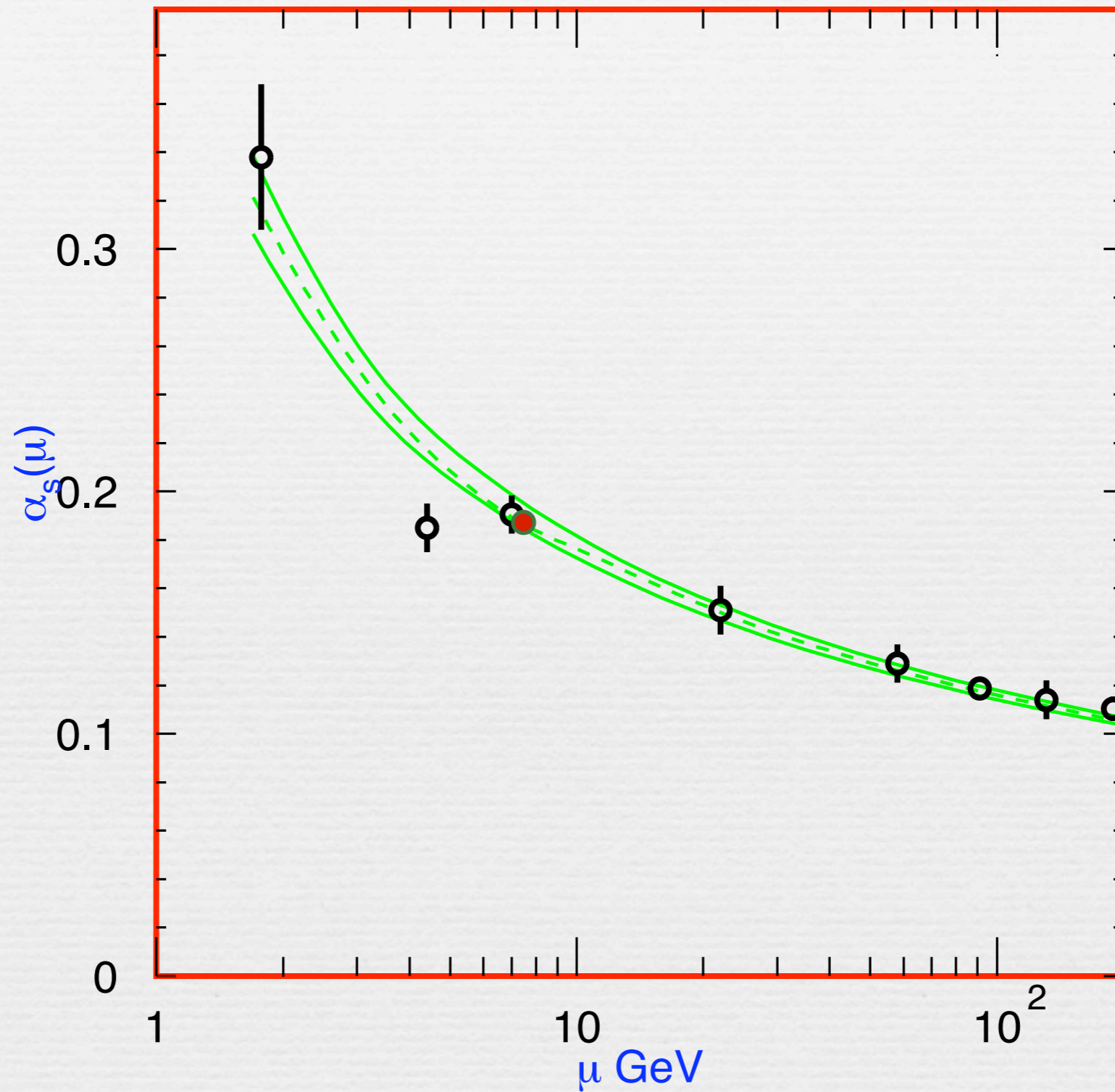
$\alpha_s = 0.1183(8)$, HPQCD, arXiv:0807.1687;

$\alpha_s = 0.1192(11)$, Maltman, arXiv:0807.2020;

$\alpha_s = 0.1185(9)$, PDG non-lat average (2008).



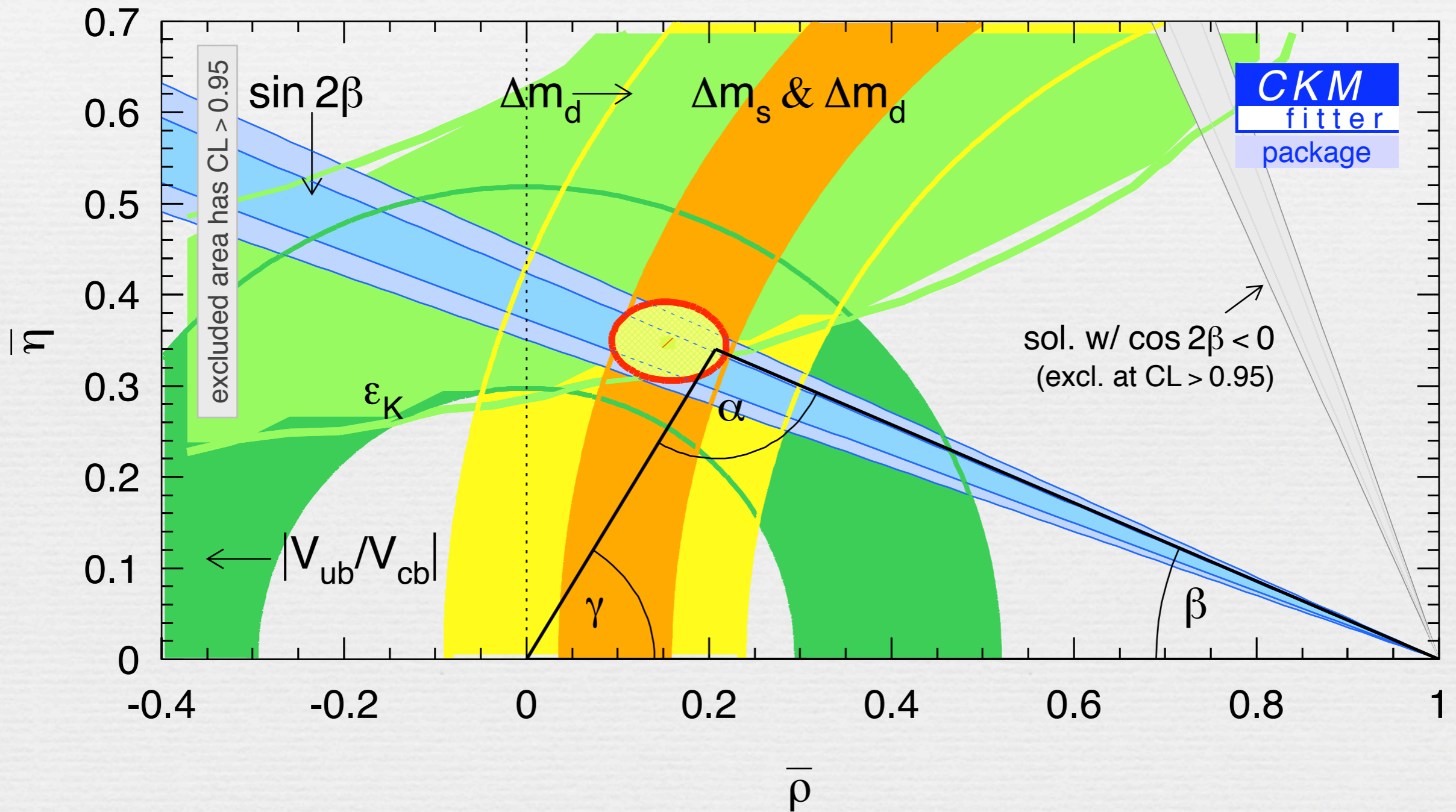
PDG 2008



QCD of hadrons = QCD of partons

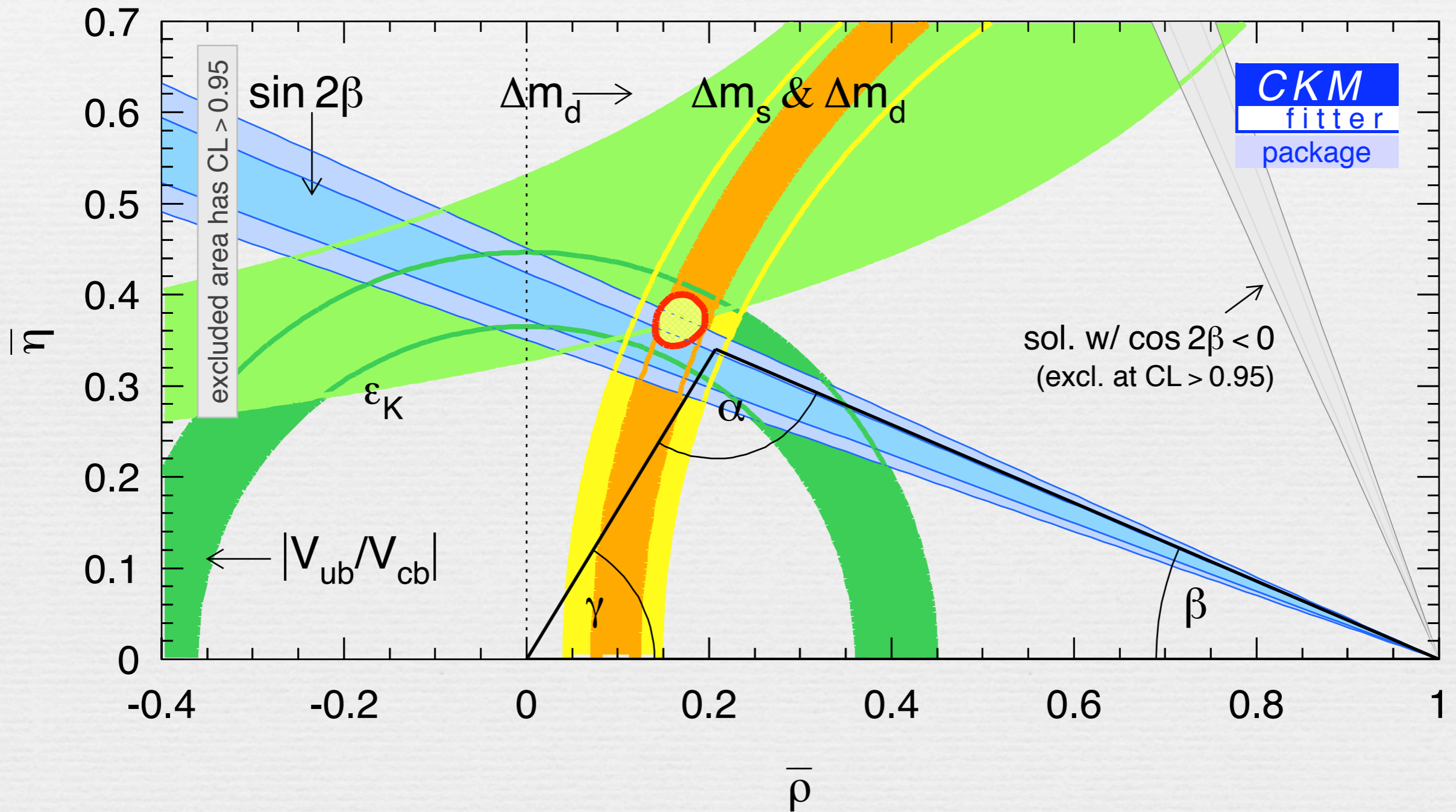
Flavor Physics

CKM UT Now



Plot from Ruth Van de Water

CKM UT 2014



Plot from Ruth Van de Water

Scope of this talk

- Neutral meson mixing: K, B, B_s .
- Semileptonic form factors:
 - $K \rightarrow \pi l \nu$ for $|V_{us}|$: RBC+UKQCD, 2007
 - $D \rightarrow K l \nu, D \rightarrow \pi l \nu$: Fermilab+MILC, 2004
 - $B \rightarrow D^* l \nu$ for $|V_{cb}|$; $B \rightarrow \pi l \nu$ for $|V_{ub}|$
- Leptonic decay constants: $f_\pi, f_K, f_D, f_{D_s}, f_B$.

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$|V_{cb}|$

Alia et Jack Laiho et al., arXiv:0808.2519

- $|V_{us}|$, $|V_{ub}|$, and $|V_{cb}|$ are the three real parameters of the CKM matrix.
- $|V_{cb}|$ normalizes the unitarity triangle: enters all flavor physics.
- Inclusive $b \rightarrow cl\nu$: OPE + PT + measured moments.
- Exclusive $B \rightarrow D^* l\nu$: (zero recoil) form factor:

$$\mathcal{F}(1) = h_{A_1}(1), \quad \langle D^* | \mathcal{A}_\mu | B \rangle = i\sqrt{2m_{D^*}2m_B} \bar{\epsilon}_\mu^* h_{A_1}(1)$$

- Previous quenched calculation (2001):

$$\mathcal{F}(1) = 0.913_{-0.017}^{+0.024} \pm 0.016_{-0.014}^{+0.003} + 0.000 + 0.006_{-0.014}$$

stats match a χ PT $n_f=0$

used till now with HFAG $|V_{cb}| \mathcal{F}(1)$ to get $|V_{cb}|$.

- Three double ratios, devised so that all uncertainties scale with $\mathcal{F}-1$, not \mathcal{F} .
- Update to 2+1 sea quarks with a *single* ratio — more direct & much less computer time.

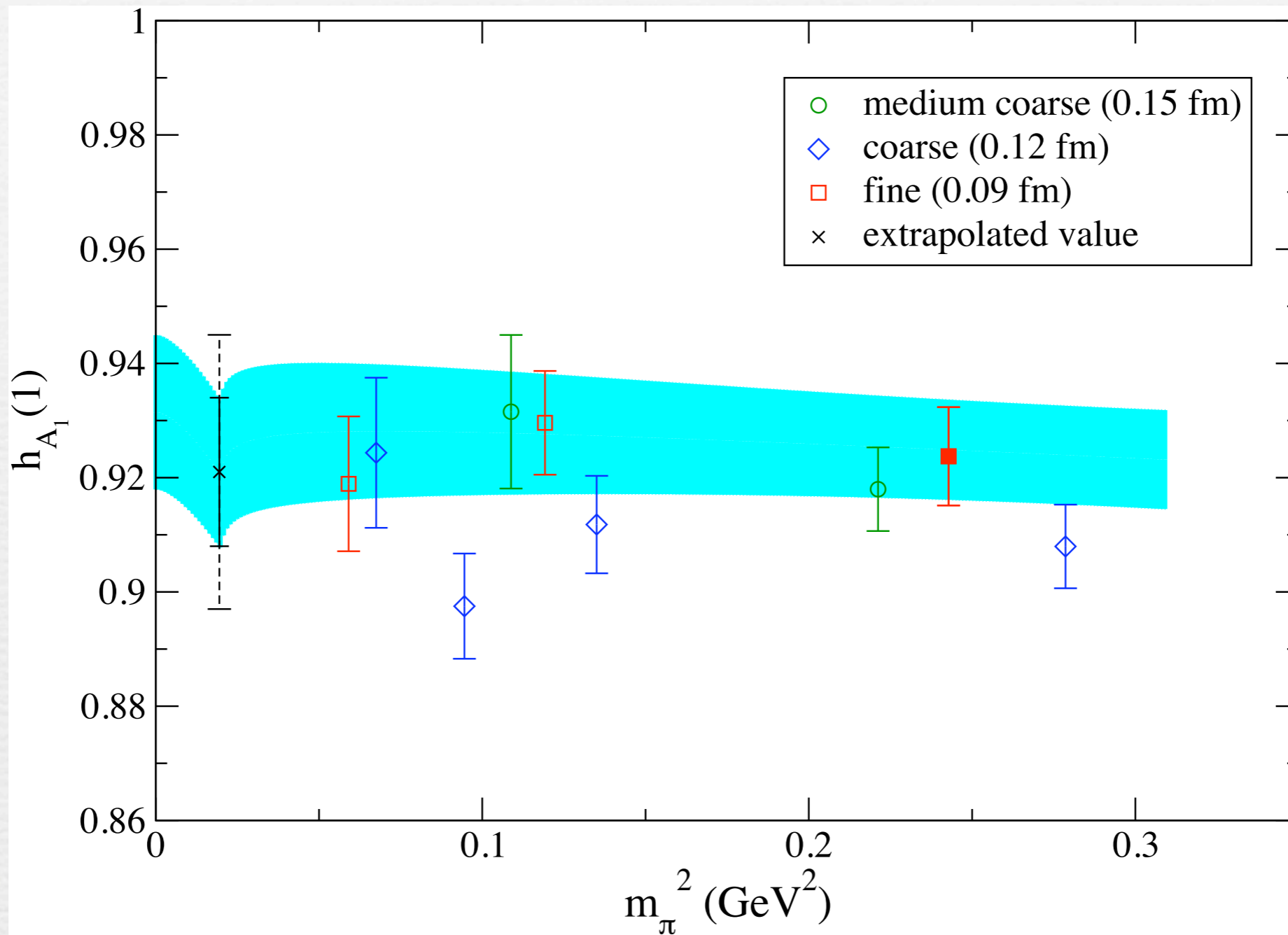
- Also introduces ratios of matrix elements to disentangle chiral extrapolation from heavy-quark discretization effects:

$$\mathcal{R}_{\text{val}}(m_x, \hat{m}', m'_s, a) := \frac{h_{A_1}(m_x, \hat{m}', m'_s, a)}{h_{A_1}(m_x^{\text{fid}}, \hat{m}', m'_s, a)},$$

$$\mathcal{R}_{\text{sea}}(\hat{m}', m'_s, a) := \frac{h_{A_1}(m_x^{\text{fid}}, \hat{m}', m'_s, a)}{h_{A_1}(m_x^{\text{fid}}, \hat{m}^{\text{fid}}, m_s^{\text{fid}}, a)}.$$

- Reconstruct

$$h_{A_1} = h_{A_1}(m_x^{\text{fid}}, \hat{m}^{\text{fid}}, m_s^{\text{fid}}, a \rightarrow 0) \\ \times \mathcal{R}_{\text{val}}(m_x, \hat{m}', m'_s, a) \times \mathcal{R}_{\text{sea}}(\hat{m}', m'_s, a)$$



$$\mathcal{F}(1) = 0.921 \pm 0.013 \pm 0.008 \pm 0.008 \pm 0.014 \pm 0.007$$

stats
 $g_{D^*D\pi}$
 χ PT
match
 m_Q

$|V_{ub}|$

alia et Ruth Van de Water, arXiv:0811.3640

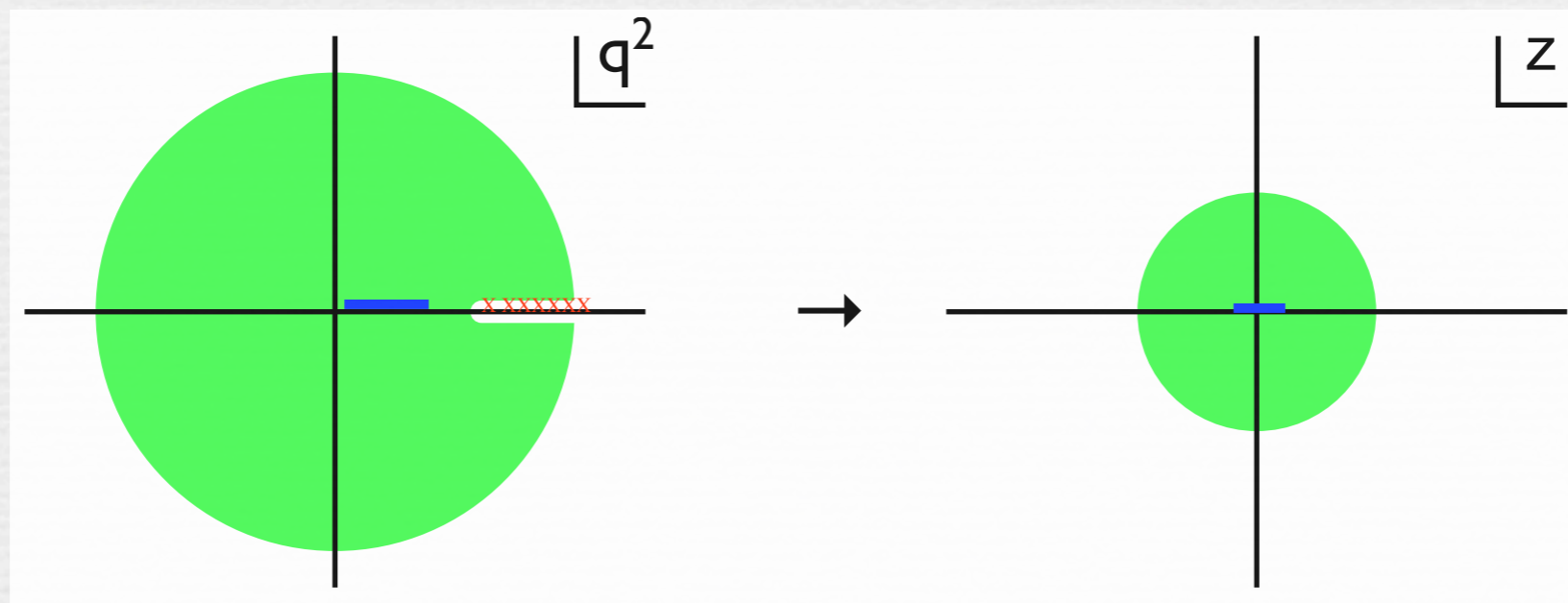
- $|V_{us}|$, $|V_{ub}|$, and $|V_{cb}|$ are the three real parameters of the CKM matrix.
- $|V_{ub}|$ gives a tree constraint comparable to $\sin 2\beta$.
- Inclusive $b \rightarrow ul\nu$: keep control of OPE (or shape functions, or ...) in region with no charm.
- Exclusive $B \rightarrow \pi l\nu$: form factor $f_+(q^2)$

$$\langle \pi | \mathcal{V}_\perp^\mu | B \rangle = (p_B + p_\pi)_\perp^\mu f_+(q^2), \quad q \cdot p_\perp = 0$$

- Problem to determine $|V_{ub}|$:
 - lattice best when p_π small, so $q^2 \approx q^2_{\max}$,
 - but event rate highest when $q^2 \approx 0$.
- Until now: find least bad q^2 of both worlds, or introduce *Ansatz* for q^2 dependence.
- Here: a model *independent* simultaneous fit.

\curvearrowright Let $z = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}},$ $t_{\pm} = (m_B \pm m_{\pi})^2$
 $t_- < t_0 < t_+$

inspired by unitarity.



\curvearrowright For $B \rightarrow \pi/\nu$ kinematics $-0.34 < z < 0.22$.

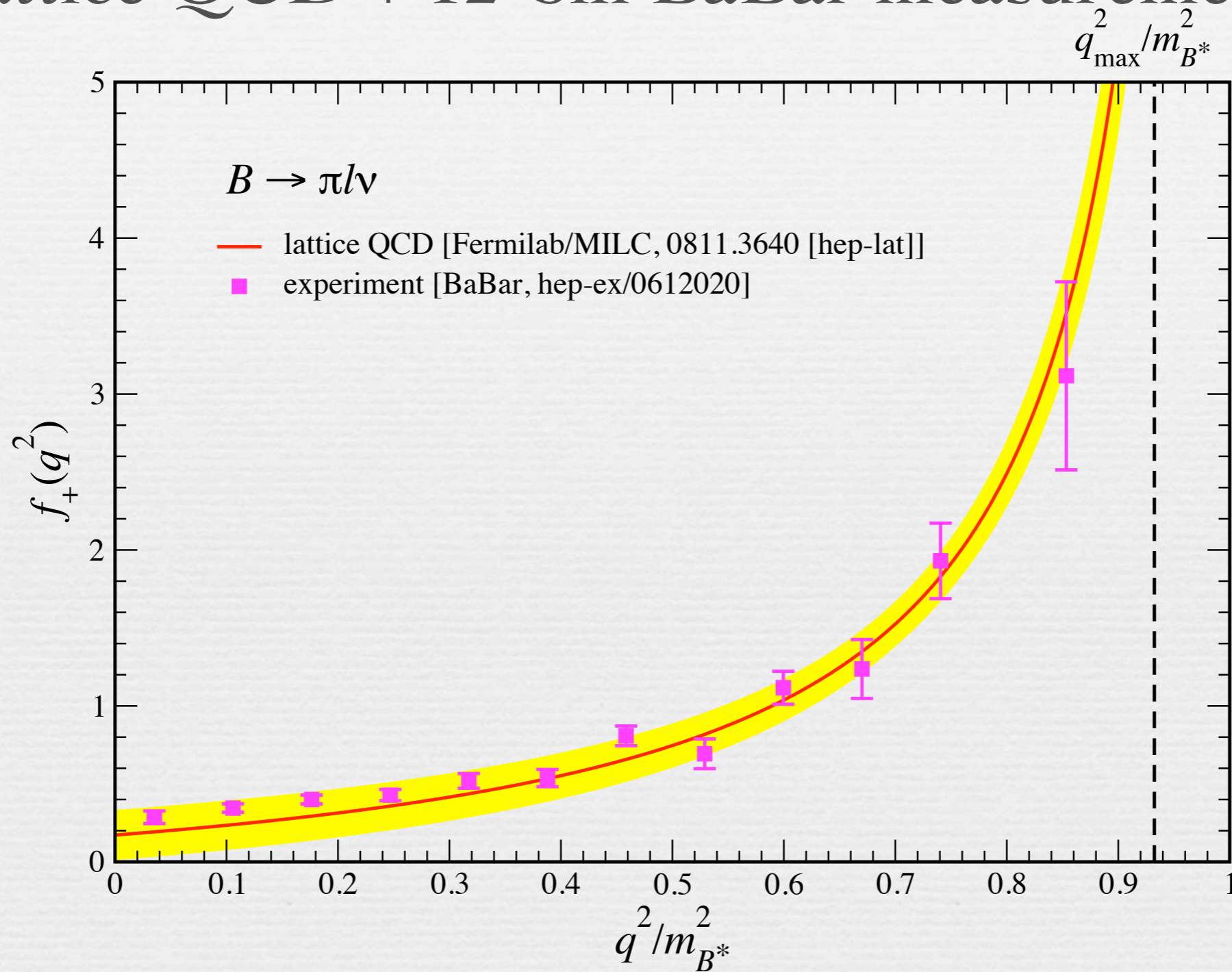
- Unitarity guarantees convergent expansion in $z(t)$:

$$P(t)\phi(t, t_0)f(t) = \sum_{k=0}^{\infty} a_k z^k, \quad \sum_{k=0}^N a_k^2 \leq 1$$

B^* pole
asymptotics

- New approach
 - fit lattice & expt **separately**: compare a_k/a_0 ;
 - fit lattice & expt **together**, yielding $|V_{ub}|$.

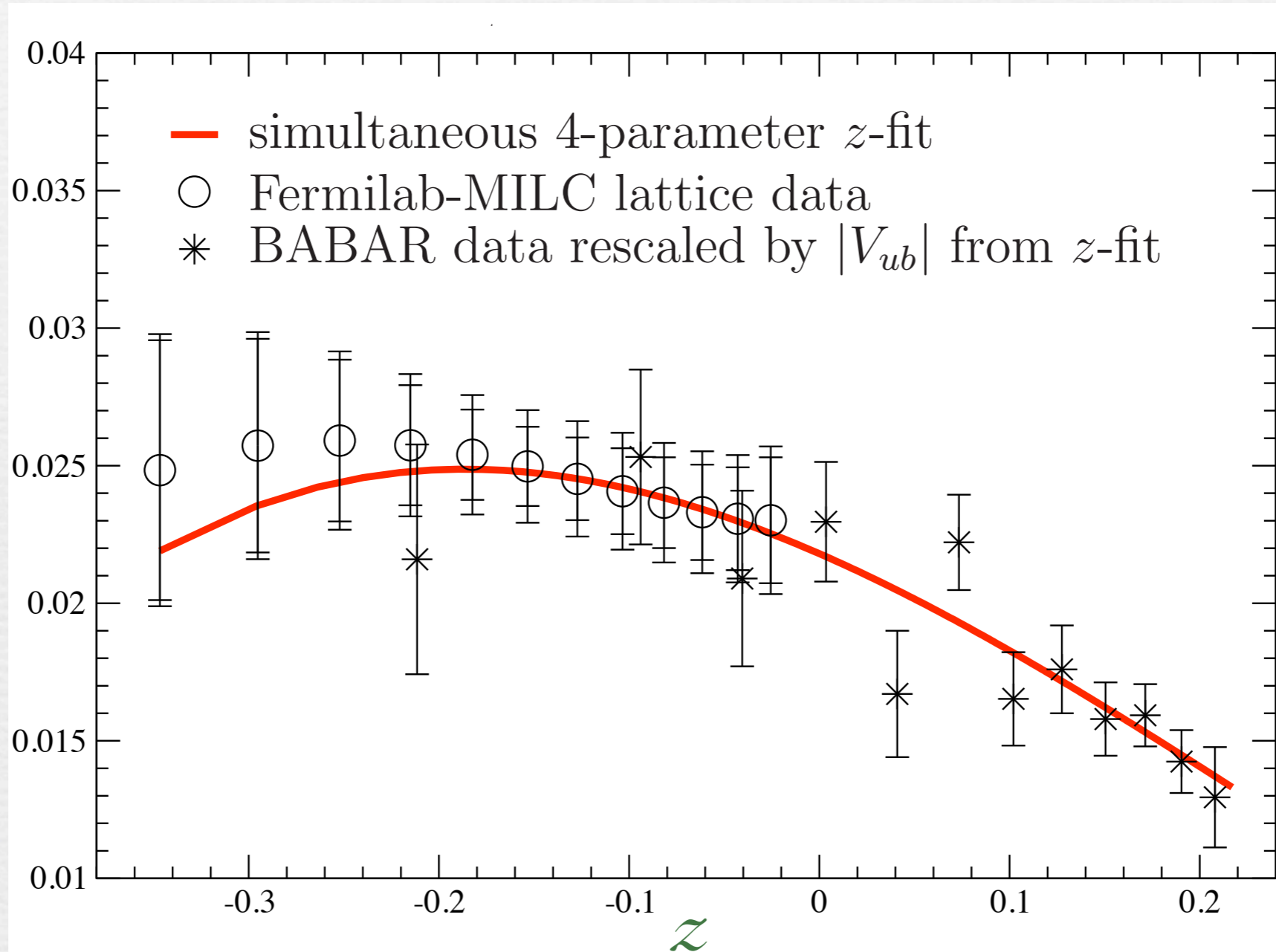
Lattice QCD + 12-bin BaBar measurement.



Fermilab Lattice + MILC

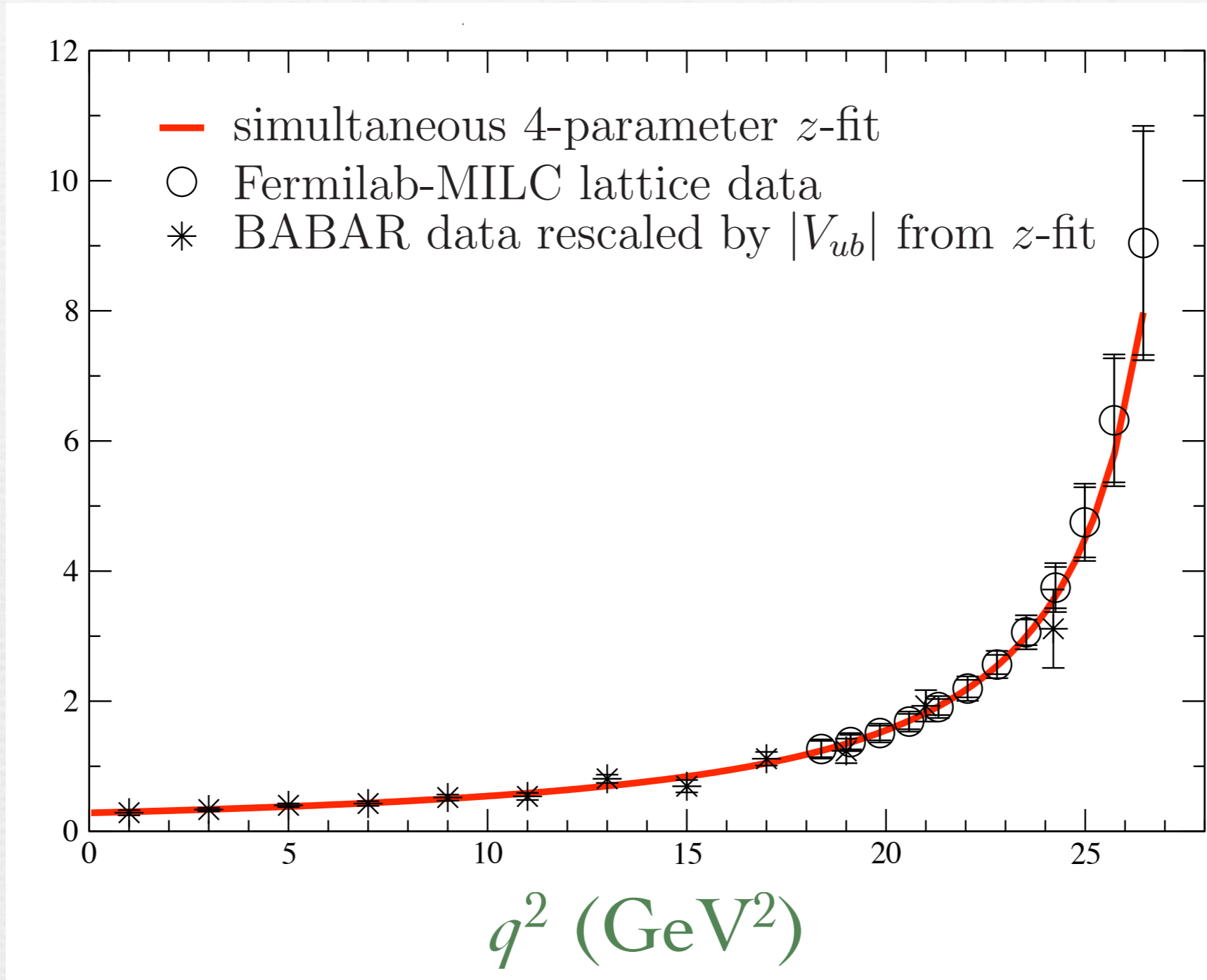
Lattice QCD + 12-bin BaBar measurement.

$P\phi f_+$



4 fit parameters: $|V_{ub}|$, a_0 , a_1 , a_2 .

f_+



Fermilab Lattice + MILC

$|V_{cb}|$ & $|V_{ub}|$

• Using $\mathcal{F}(1)$ to get $|V_{cb}|$:

$$10^3 |V_{cb}| = 38.7(9)(10)$$

with latest HFAG.

• Compared to inclusive:

$$10^3 |V_{cb}| = 41.6(8)$$

from HFAG/ICHEP08.

• **Final** z -fit to get $|V_{ub}|$:

$$10^3 |V_{ub}| = 3.38(36)$$

with BaBar 12-bin data.

• Compared to inclusive:

$$10^3 |V_{ub}| = (3.76-4.87) \pm 0.35$$

from HFAG/ICHEP08.

Being sorted out for CKM 2008 report.

f_B and B_B

Elvira Gamiz *et al.* [HPQCD], arXiv:0902.1815

- Recent 2+1 calculation of neutral B mixing:

$$\begin{aligned} f_{B_s} &= 231 \pm 15 \text{ MeV} & f_{B_s} \sqrt{B_{B_s}} &= 266 \pm 18 \text{ MeV} \\ f_{B_d} &= 190 \pm 13 \text{ MeV} & f_{B_d} \sqrt{B_{B_d}} &= 216 \pm 15 \text{ MeV} \\ & & \xi &= 1.258 \pm 0.033 \end{aligned}$$

- Decay constants 1σ lower than in hep-lat/0507015, stemming from 2nd, finer lattice spacing.
- Fermilab/MILC (Lat'08 preliminary):
 $f_B = 195 \pm 11 \text{ MeV}$, $f_{B_s} = 243 \pm 11 \text{ MeV}$.

f_{D_s} Puzzle

f_π and f_K

- These are thought of as tests of (lattice) QCD.
- Experiments yield $|V_{ud}|f_\pi$ and $|V_{us}|f_K$:
 - $|V_{ud}|$ from nuclear $0^+ \rightarrow 0^+$ transitions;
 - $|V_{us}|$ from semileptonic $K \rightarrow \pi/\nu$.
- Many groups (MILC, RBC, NPLQCD, Aubin-Laiho-Van de Water, HPQCD, BMW) have these (or ratio f_K/f_π) with accuracy at 2% level.

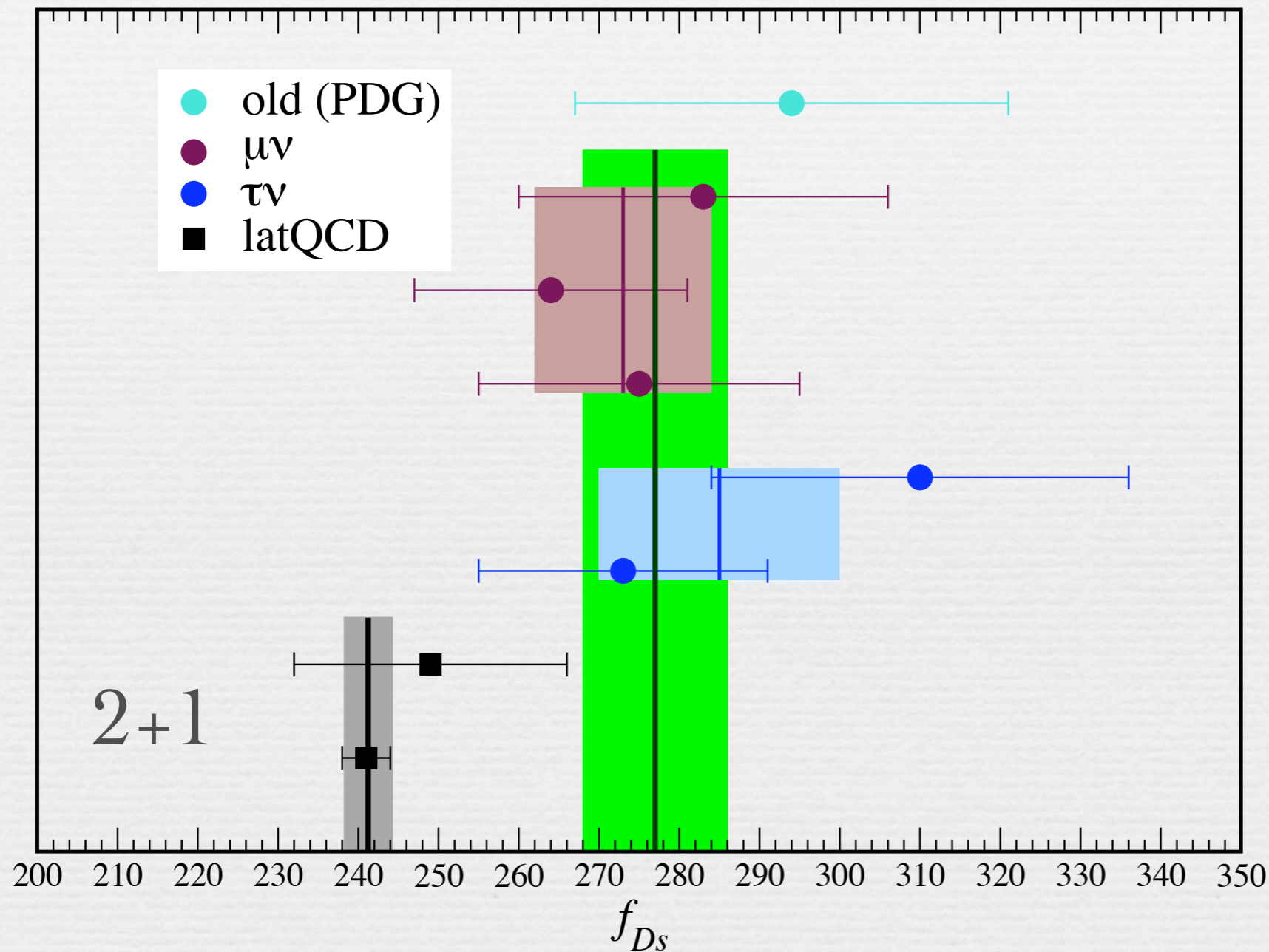
f_D and f_{D_s}

- These are thought of as tests of (lattice) QCD.
- Experiments (recently) yield $|V_{cd}|f_D$ and $|V_{cs}|f_{D_s}$:
 - $|V_{cx}|$ from CKM unitarity.
- First *unquenched* calculations [Fermilab/MILC] agreed, at 7% level, with first good measurements (CLEO for D , BaBar for D_s).

$$D_s \rightarrow l\nu$$

- $D_s \rightarrow l\nu$ should be the easiest leptonic decay for lattice QCD.
- A simple matrix element $\langle 0 | \bar{s} \gamma_\mu \gamma_5 c | D_s \rangle = i f_{D_s} p_\mu$.
- No light valence quarks.
- Counting experiment at CLEO, B factories.
- New physics thought to be *very unlikely*.

And then something funny happened (end 2007)...



$\chi^2/\text{dof} = 0.67$

BaBar

CLEO

Belle

CLEO $\pi\nu$

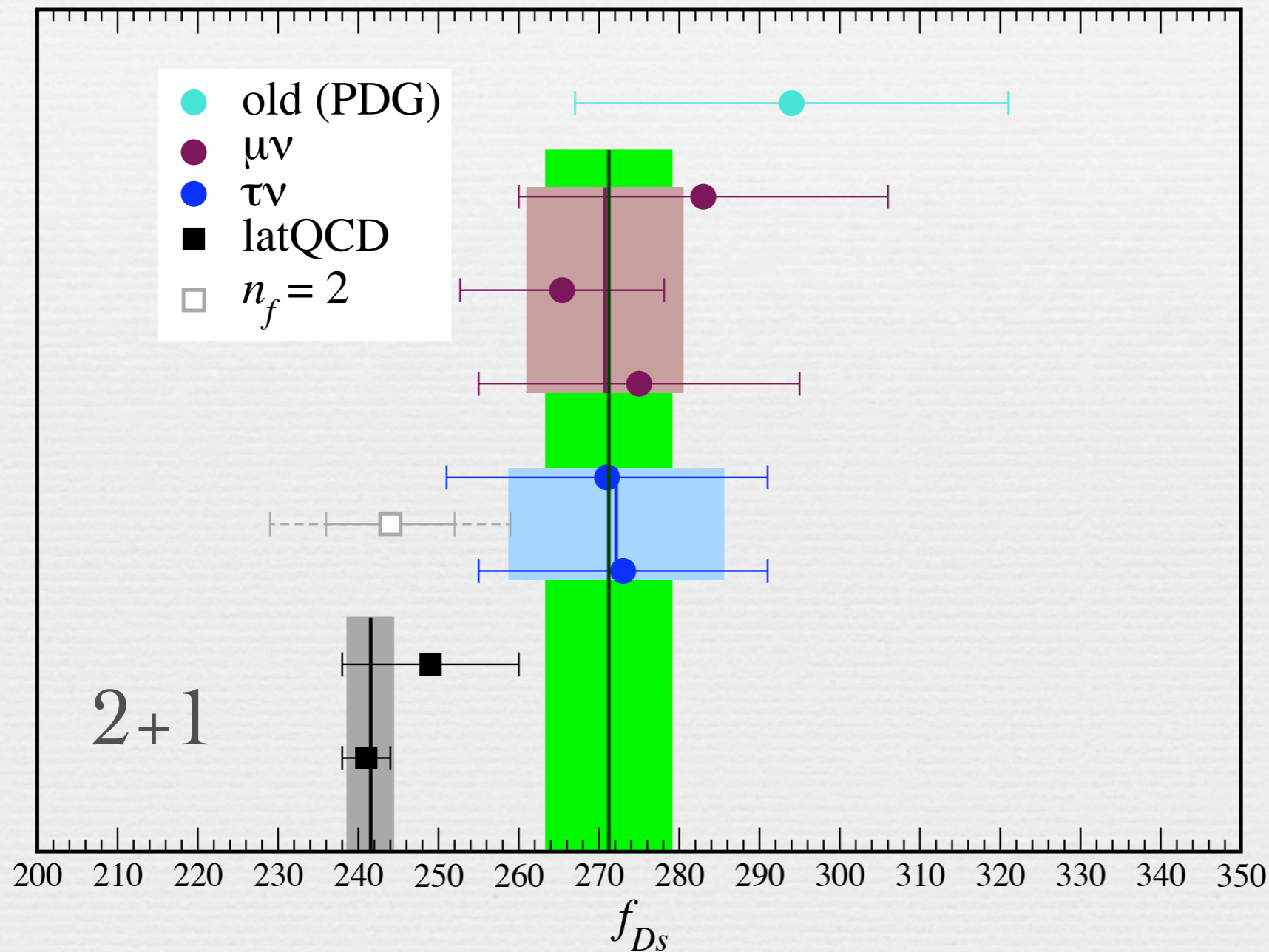
CLEO $e\nu\nu$

Fermilab/MILC

HPQCD

a 3.8σ discrepancy, or $2.7\sigma \oplus 2.9\sigma$.

Updates from FPCP (CLEO) and Lat'08 ...



$\chi^2/\text{dof} = 0.13$

BaBar

CLEO

Belle

CLEO $\pi\nu$

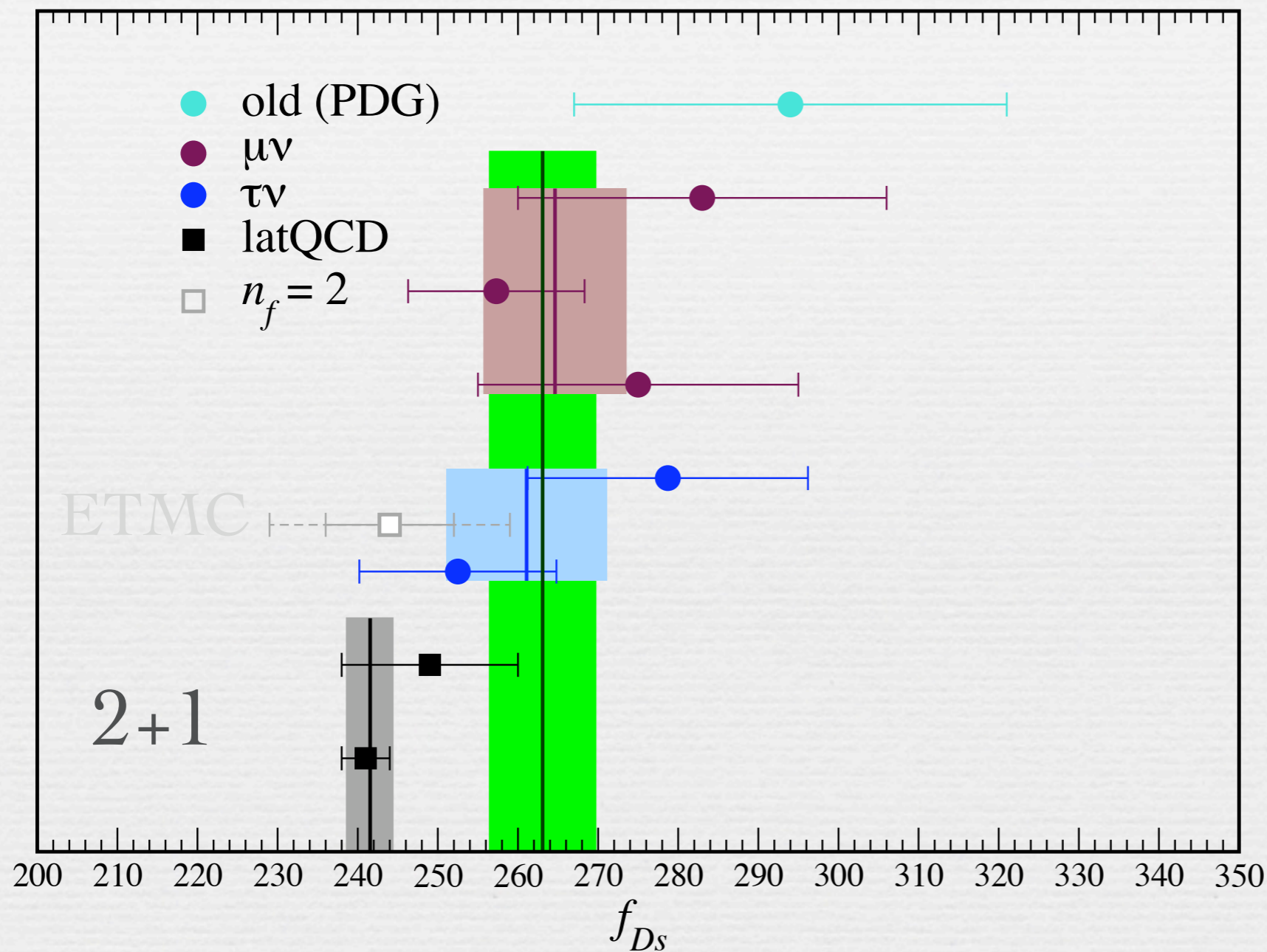
CLEO $e\nu\nu$

Fermilab/MILC

HPQCD

a 3.6σ discrepancy, or $2.9\sigma \oplus 2.2\sigma$.

With CLEO's papers of January 12, 2009



$\chi^2/\text{dof} = 0.73$

BaBar

CLEO

Belle

CLEO $\pi\nu$

CLEO $e\nu\nu$

Fermilab/MILC

HPQCD

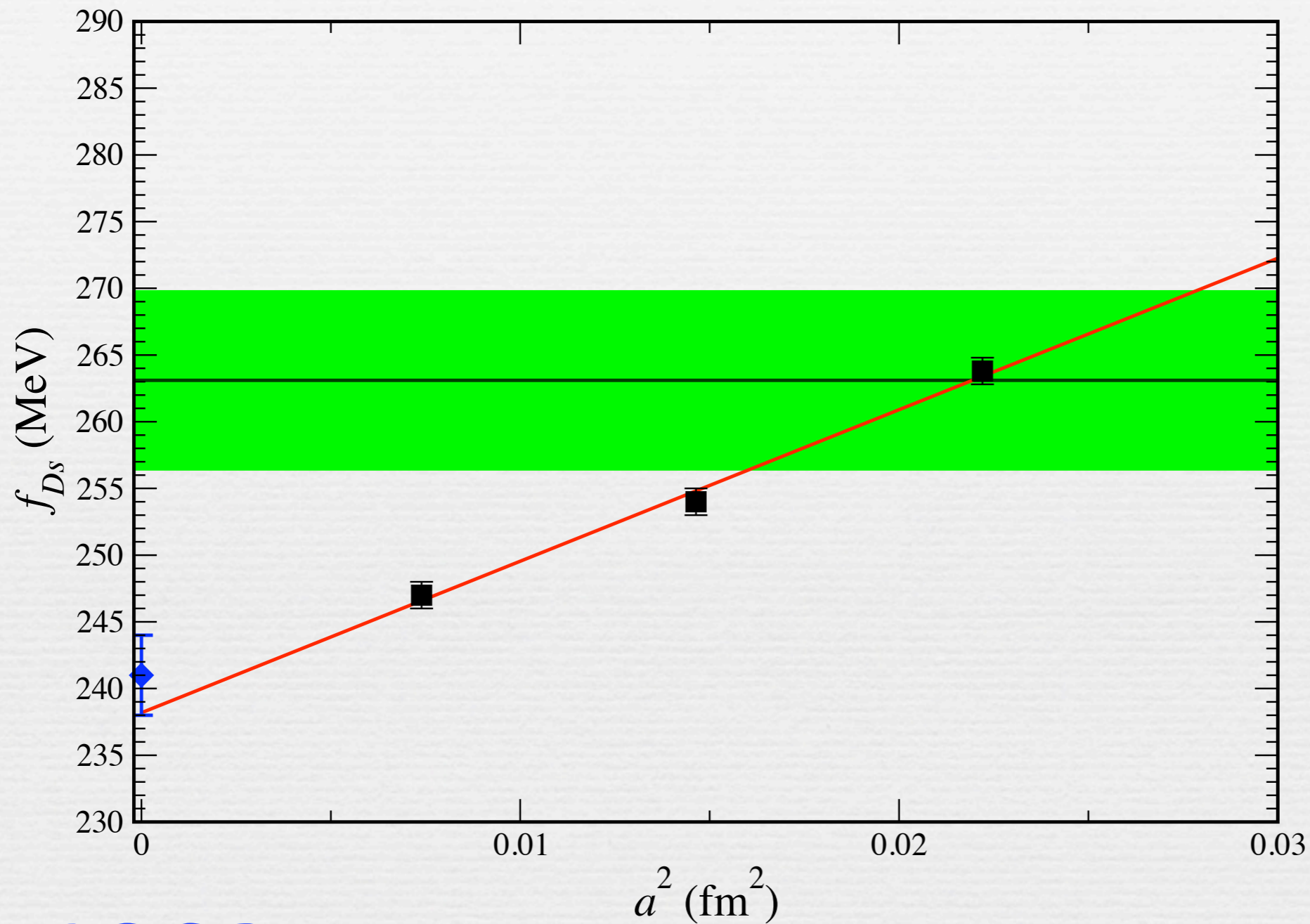
a 3.0σ discrepancy, or $2.5\sigma \oplus 1.9\sigma$.

A Puzzle

$$B(D_s \rightarrow \ell \nu) = \frac{m_{D_s} \tau_{D_s}}{8\pi} f_{D_s}^2 |G_F V_{cs}^* m_\ell|^2 \left(1 - \frac{m_\ell^2}{m_{D_s}^2}\right)^2$$

- Experimental errors?
- Radiative corrections?
- CKM?
- Lattice QCD?
- Unlikely: stats limited.
- No: 1–2%
- No: need $|V_{cs}| > 1.1$.
- Let's see.

As the lattice gets finer, the discrepancy grows:

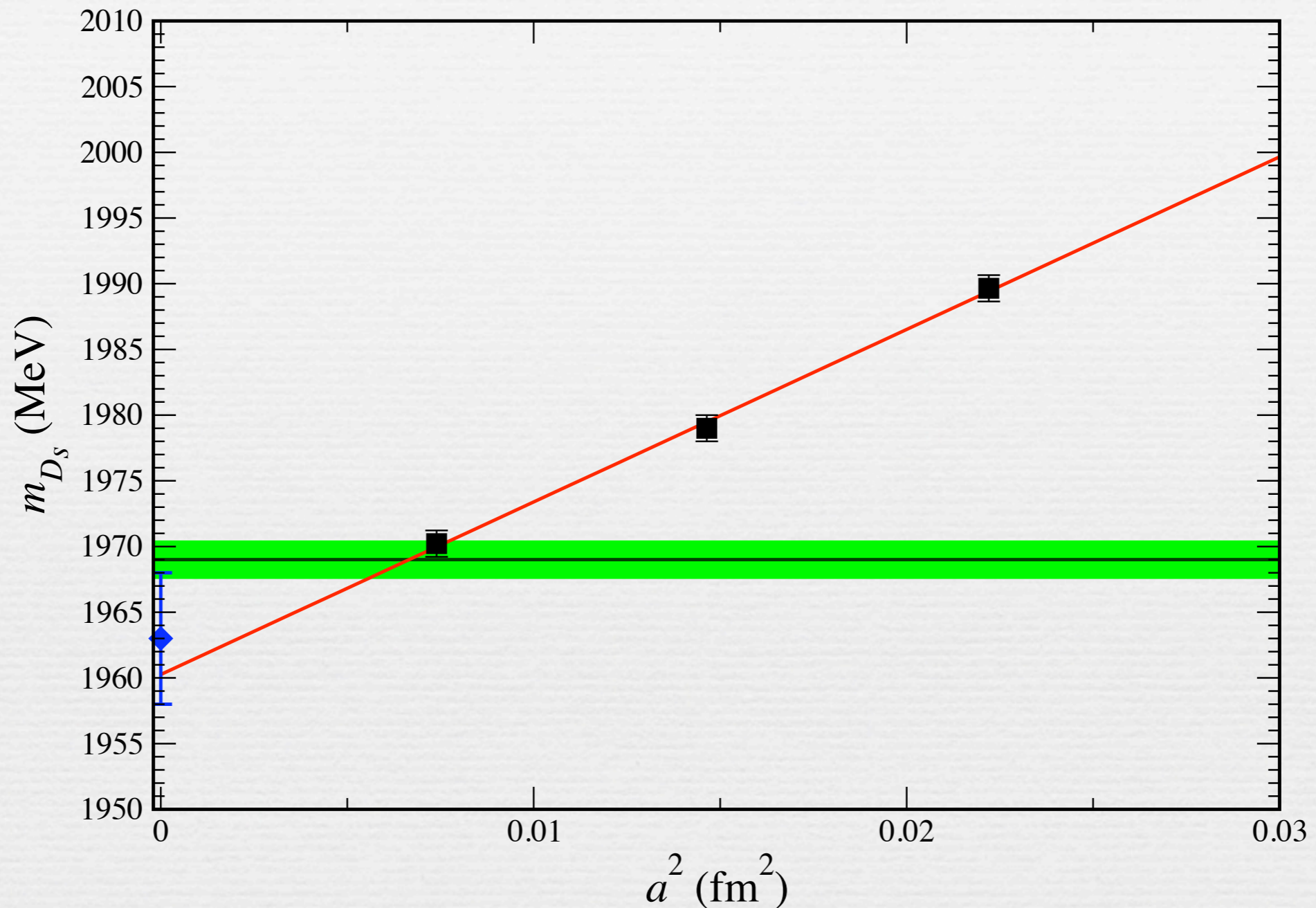


263.1 ± 6.7
MeV

slope is
 $O(\alpha_s m_c \Lambda a^2)$
as expected

HPQCD
 241 ± 3

linear in a^2 : 239; quad in a^2 : 242;
linear in a^4 : 245.



If m_c (set from η_c) were retuned to flatten this,
 f_{D_s} (at $a \neq 0$) would not change much.

Error Budget

$$\Delta_q = 2m_{Dq} - m_{\eta c}$$

	f_K/f_π	f_K	f_π	f_{D_s}/f_D	f_{D_s}	f_D	Δ_s/Δ_d
r_1 uncertainty.	0.3	1.1	1.4	0.4	1.0	1.4	0.7
a^2 extrap.	0.2	0.2	0.2	0.4	0.5	0.6	0.5
Finite vol.	0.4	0.4	0.8	0.3	0.1	0.3	0.1
$m_{u/d}$ extrap.	0.2	0.3	0.4	0.2	0.3	0.4	0.2
Stat. errors	0.2	0.4	0.5	0.5	0.6	0.7	0.6
m_s evoln.	0.1	0.1	0.1	0.3	0.3	0.3	0.5
m_d , QED, etc.	0.0	0.0	0.0	0.1	0.0	0.1	0.5
Total %	0.6	1.3	1.7	0.9	1.3	1.8	1.2

charmed sea $\ll 1\%$?

Other Results

arXiv:hep-lat/0610092 & arXiv:0706.1726 [hep-lat]

what	expt	HPQCD	
$m_{J/\psi} - m_{\eta_c}$	118.1	$111 \pm 5^\ddagger$	MeV
m_{Dd}	1869	1868 ± 7	MeV
m_{Ds}	1968	1962 ± 6	MeV
Δ_s/Δ_d	1.260 ± 0.002	1.252 ± 0.015	
f_π	130.7 ± 0.4	132 ± 2	MeV
f_K	159.8 ± 0.5	157 ± 2	MeV
f_D	$205.8 \pm 8.9^*$	207 ± 4	MeV

*CLEO arXiv:08062112 ‡ annihilation corrected

What if

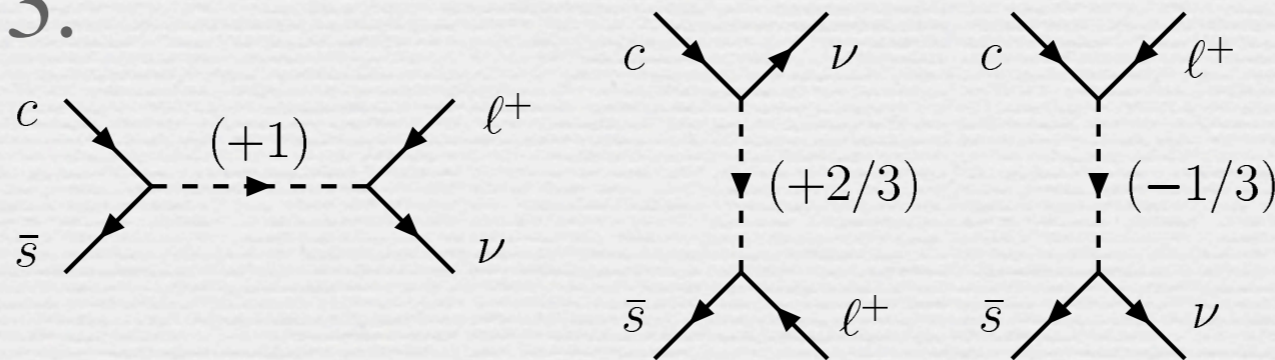
- ... the discrepancy is real?
- Then it must be non-Standard physics.
- How wacky would a non-Standard model be?
- It turns out particles that are already being considered can do the trick.
- B.A. Dobrescu & ASK, arXiv: 0803.0512

New Particles

- Effective interactions

$$\mathcal{L}_{\text{eff}} = \frac{C_A^\ell}{M^2} (\bar{s} \gamma_\mu \gamma_5 c) (\bar{\nu}_L \gamma^\mu \ell_L) + \frac{C_P^\ell}{M^2} (\bar{s} \gamma_5 c) (\bar{\nu}_L \ell_R) + \text{H.c.}$$

can be induced by heavy particles of charge +1, +2/3, -1/3.

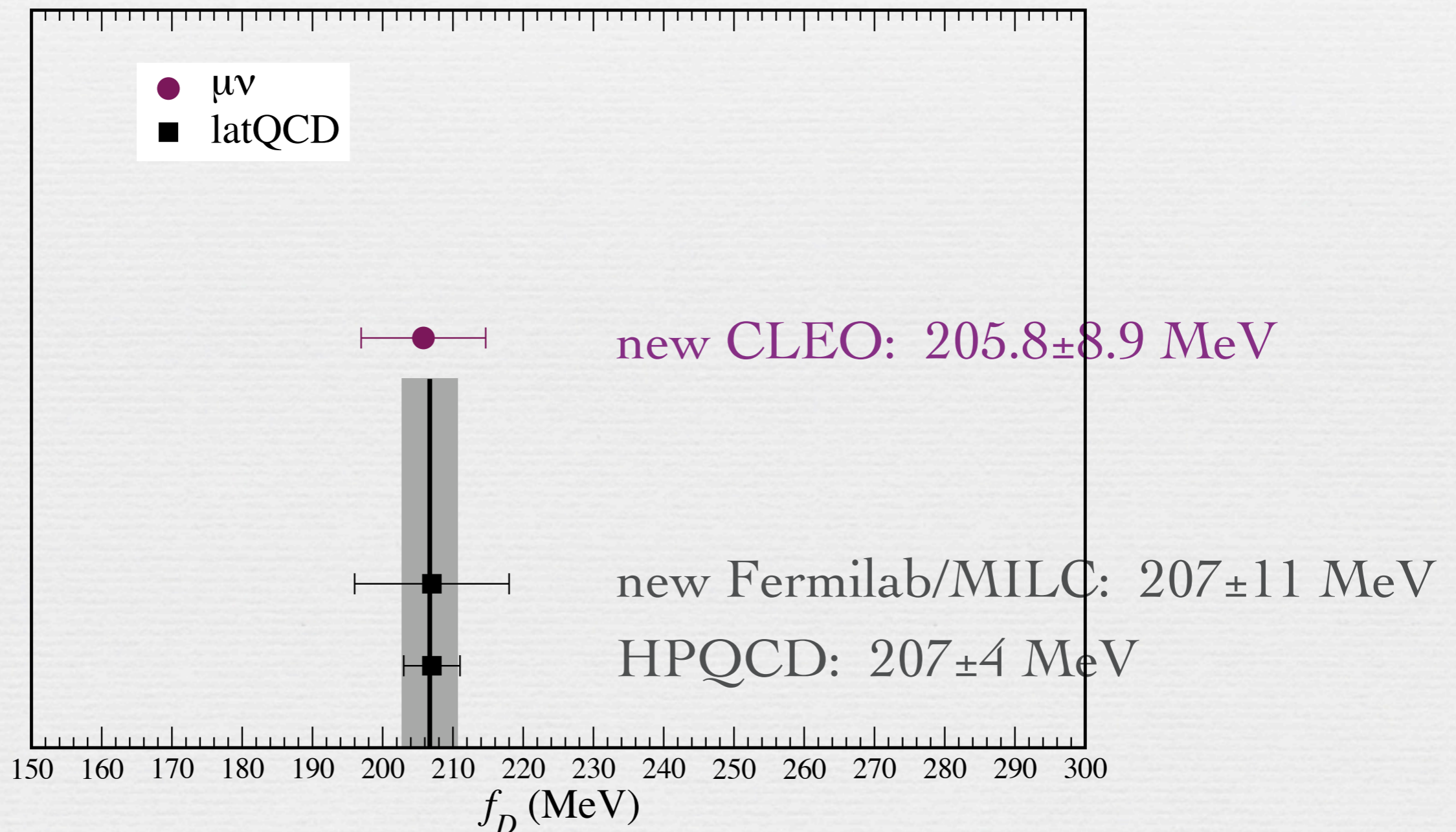


- Charged Higgs, new W' ; leptoquarks.

Beyond SM

- New W' boson: unlikely.
- Charged Higgs:
 - Model II destructively interference;
 - BAD & ASK found new model.
- Leptoquarks:
 - $J = 0, (3, 1, -1/3)$, aka \tilde{d} , can explain the effect.

- Charged Higgs model predicts a similarly-sized deviation in $D \rightarrow l\nu$, now disfavored:



LHC

- The generic bounds on mass/coupling:

$$\frac{M}{(\text{Re}C_{A,P}^{\ell})^{1/2}} \lesssim \begin{cases} 710 \text{ GeV}, & 920 \text{ GeV} \text{ for } \ell = \tau \\ 850 \text{ GeV}, & 4500 \text{ GeV} \text{ for } \ell = \mu \end{cases}$$

any non-Standard explanation of the effect is observable at the LHC.

- Leptoquarks: $gg \rightarrow \tilde{d}\tilde{d} \rightarrow \ell_1^+ \ell_2^- j_c j_c$.

Conclusions

- Lattice QCD with 2+1 staggered sea quarks has provided many results since 2003;
 - now 2+1 Wilson and DWF sea too.
- Broad, and often precise, agreement with experiment in hadron masses, quarkonium splittings, decay properties.
- Precise agreement of α_s and heavy-quark masses, where pQCD also reliable.

- The outlier is f_{D_s} , which should be *easy*:
 - valence quarks aren't light;
 - PCAC normalization.
- Experimental *statistical* error is yardstick for discrepancy: with $2\times$ (lattice error) still 2.4σ .
- CLEO done; BaBar & Belle could revisit; BES will go further in a few years.
- If new particle, LHC will make them.



Jack Laiho
called away for job interview