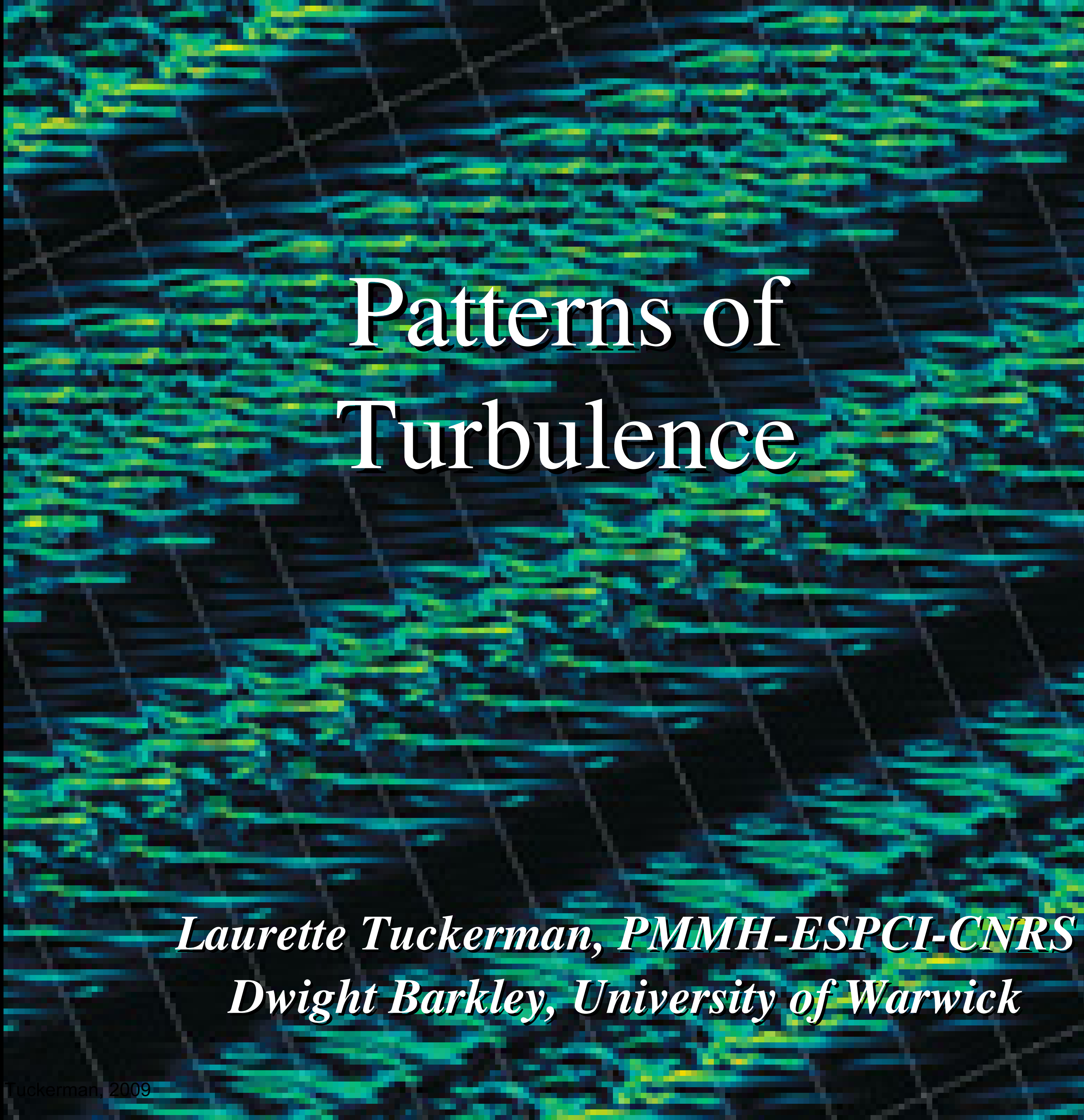
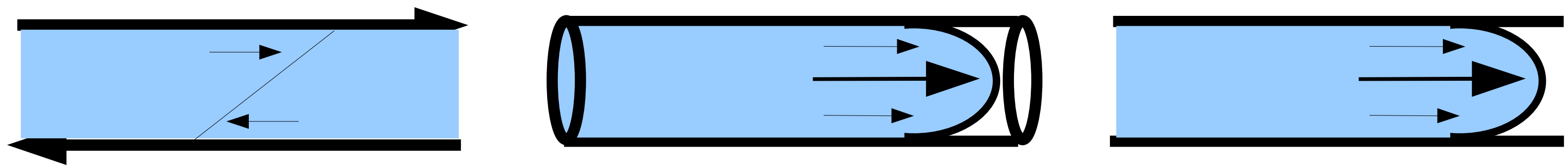


Patterns of Turbulence

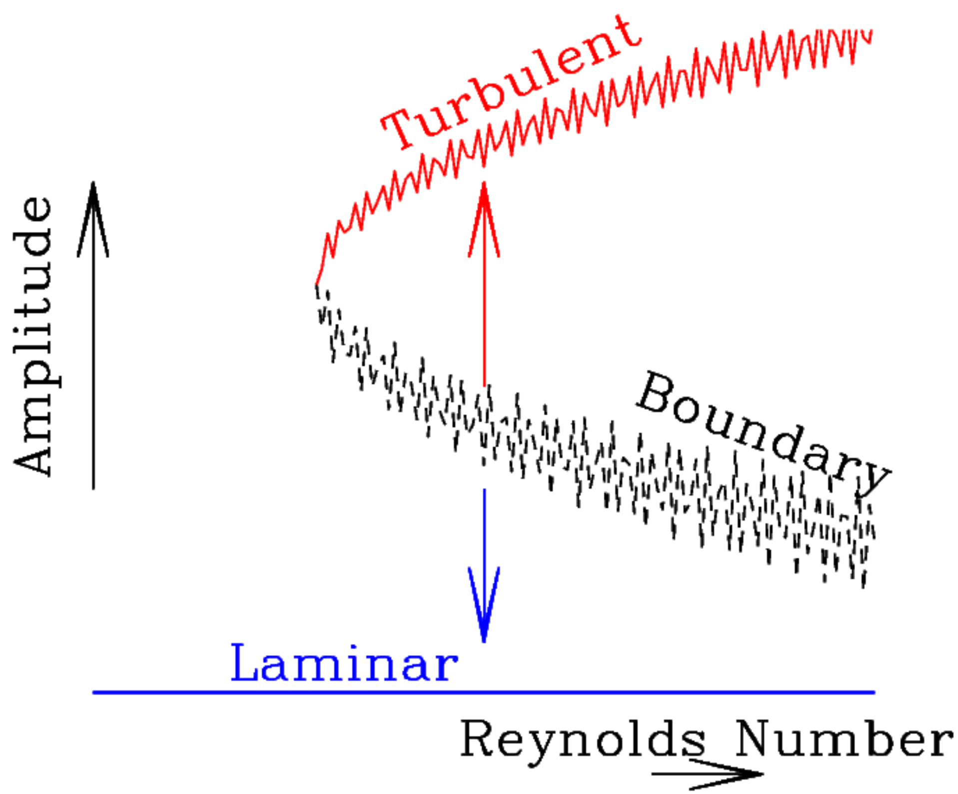
Laurette Tuckerman, PMMH-ESPCI-CNRS

Dwight Barkley, University of Warwick

Parallel Flows

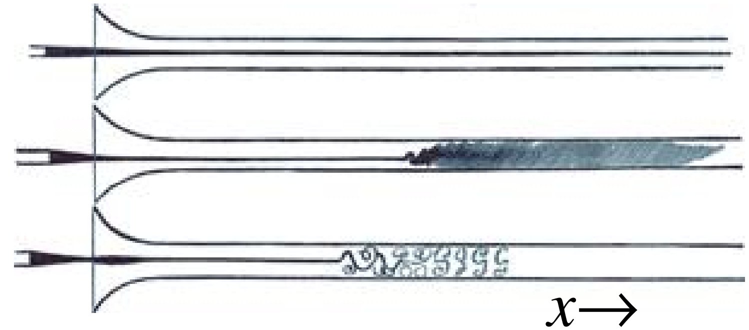


	Couette	**Pipe**	**Poiseuille**
linear instability:	$Re = \infty$ *(Romanov)*	$Re = \infty$	$Re = 5772$ *(Orszag)*
transition to turbulence:	$Re \approx 300$	$Re \approx 2000$	$Re \approx 1000$



Transition to turbulence in parallel flows

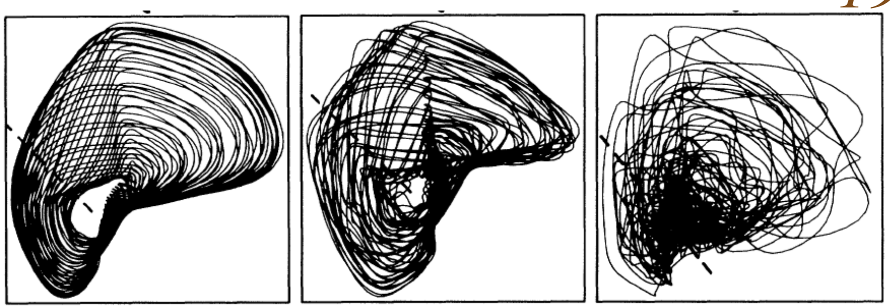
1880s ***Reynolds:*** *first systematic investigation*



1900s ***Orr-Sommerfeld equation***

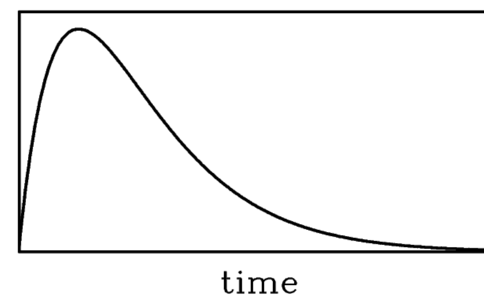
1930s ***Squire's Theorem:*** $Re_{2D} < Re_{3D}$

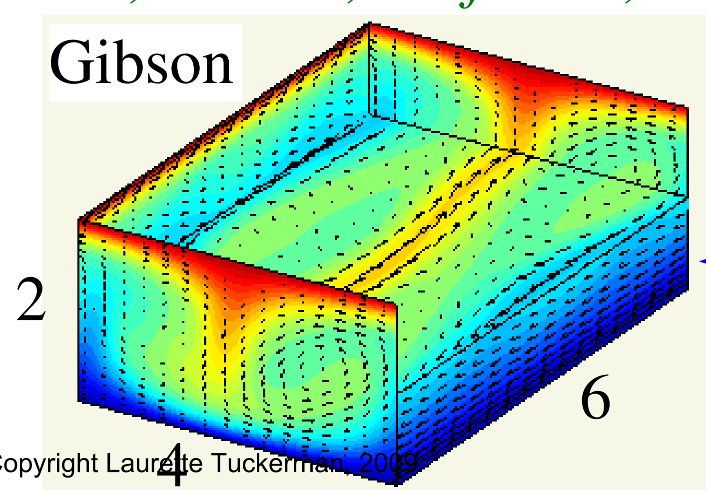
1980s ***3D instability of 2D decaying transients:***
Orszag, Patera, Kells, Bayly, Herbert



Low-dimensional chaos, strange attractors:
Swinney, Gollub, Brandstater

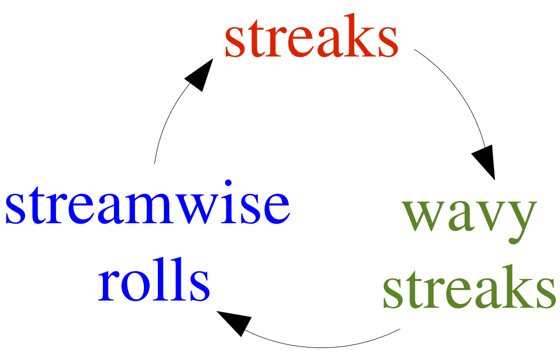
1990s ***Transient growth:***
perturbations can grow over finite time even if all $\lambda < 0$
Butler, Farrell, Trefethen, Schmid, Henningson, Reddy



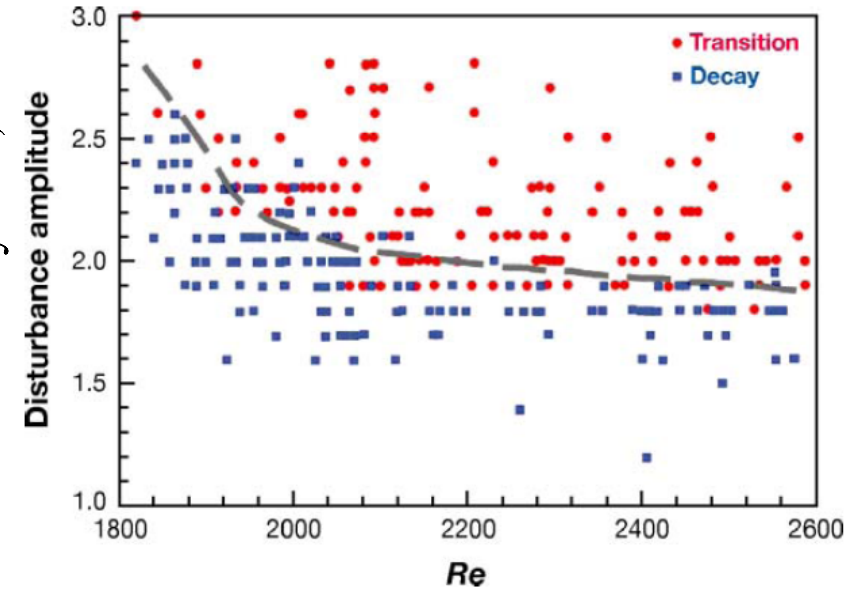


1990s ***Minimal flow unit:*** *smallest periodic boxes sustaining turbulence in simulations*
Hamilton, Kim, Waleffe, Jiménéz

Self-sustaining process: *Waleffe*



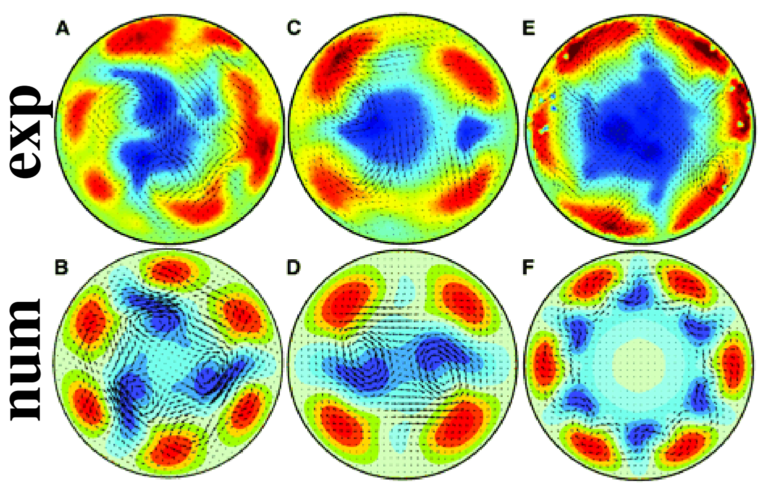
Mullin & Darbyshire, 1995



Re dependence of minimum triggering perturbations and turbulent lifetimes:

Mullin, Darbyshire, Peixinho, Hof, Daviaud, Dauchot, Manneville, Eckhardt, Faisst

Basin boundary is fractal/edge states: *Eckhardt, Schmiegel, Schneider, Yorke, Skufca*



Hof et al., 2004

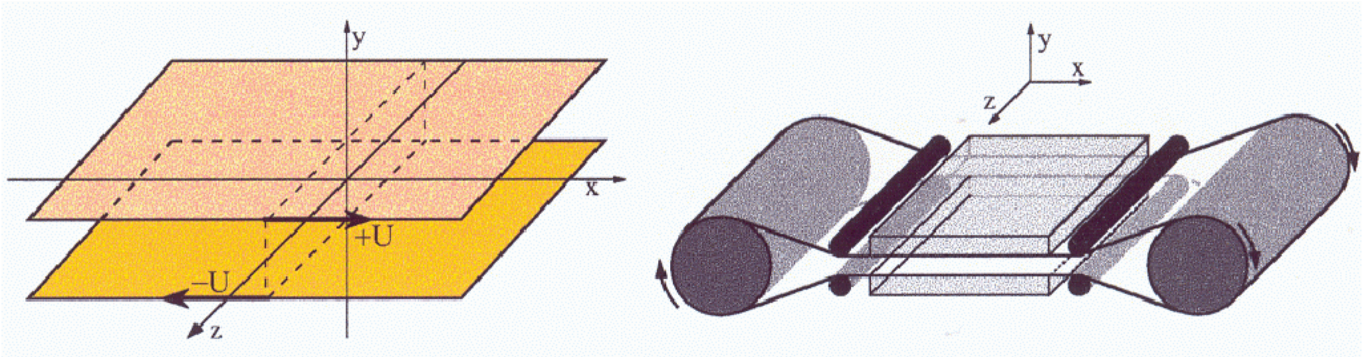
New unstable solutions form skeleton of chaotic attractor = turbulence

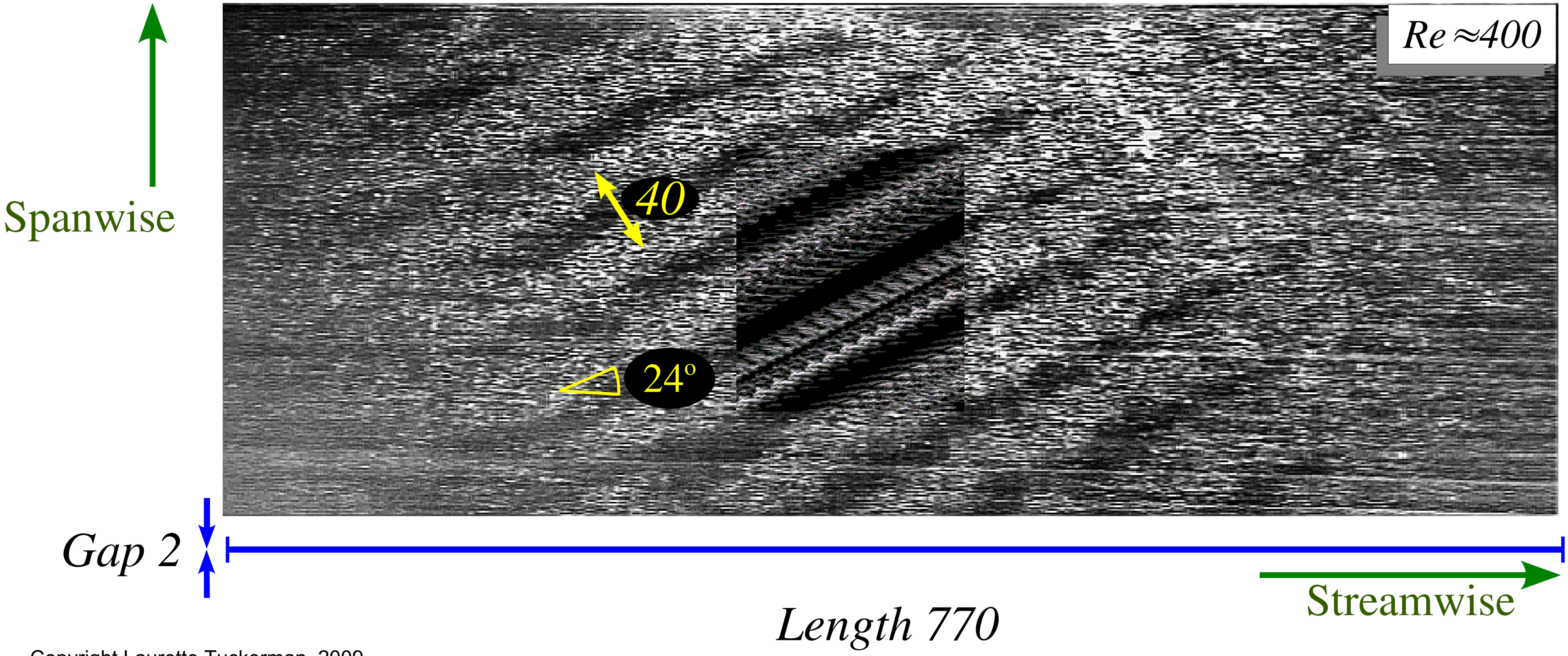
Nagata, Busse, Ehrenstein, Kawahara, Kida, Waleffe, Cvitanovic, Gibson, Halcrow, Viswanath, Kerswell, Wedin, Pringle, Duguet, Willis, Eckhardt, Faisst

Experiments at CEA/Saclay by Prigent, Dauchot (2000-3)

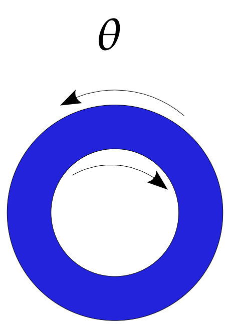
Plane Couette Flow

$$Re = \frac{U\ gap/2}{\nu}$$

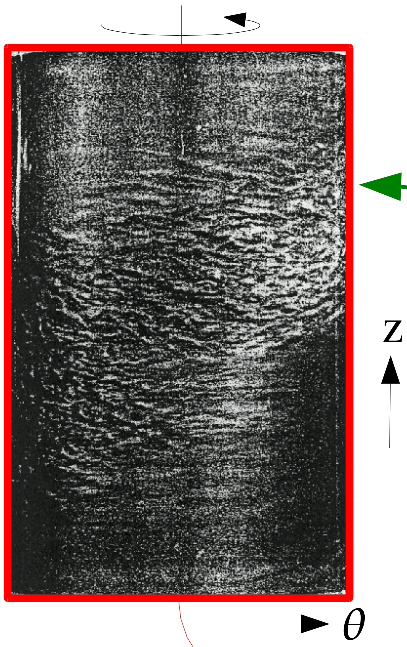




Spiral Turbulence in counter-rotating Taylor-Couette Flow

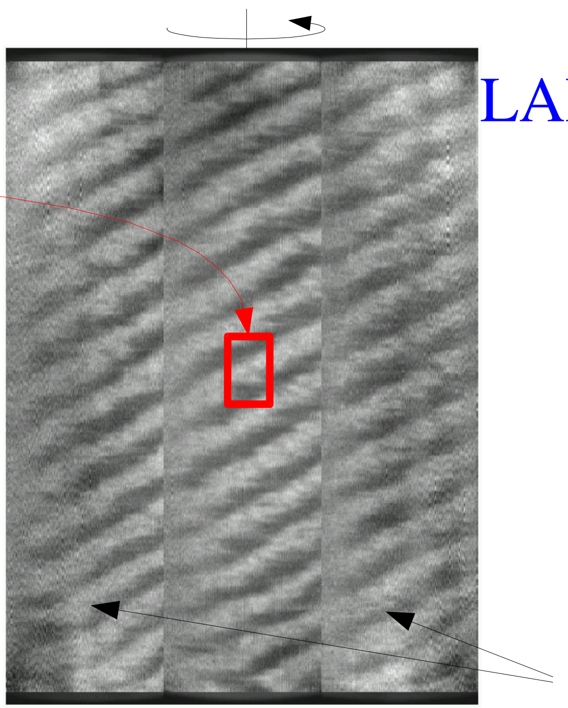


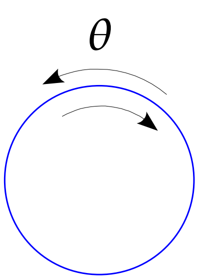
$$\eta = \frac{R_{in}}{R_{out}} = 0.88$$



- Coles JFM (1965)
- van Atta JFM (1966)
- Andereck et al. JFM (1986)

LARGE aspect ratio

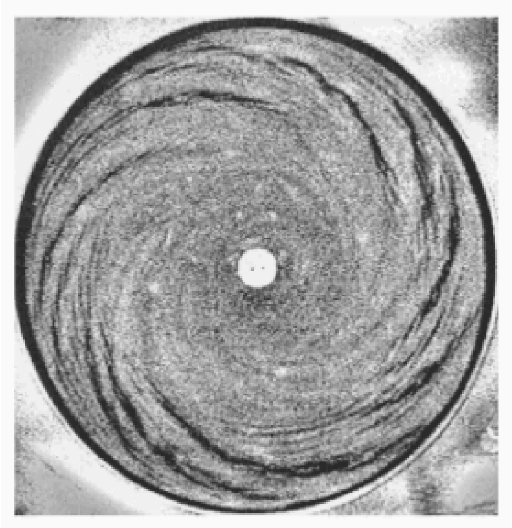




$$\eta = \frac{R_{in}}{R_{out}} = 0.983$$

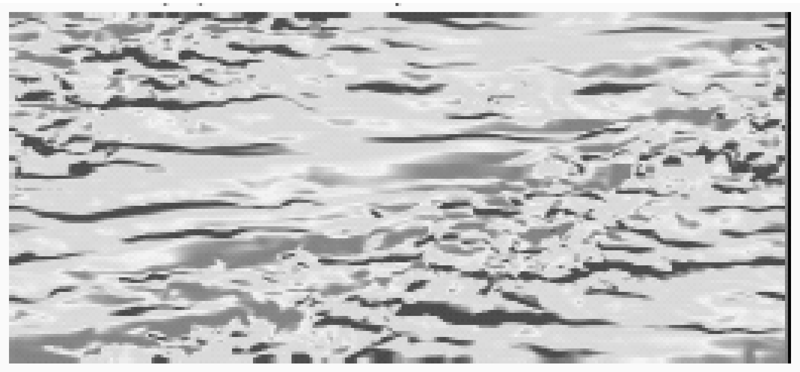
Prigent & Dauchot PRL (2002)

Rotor-Stator



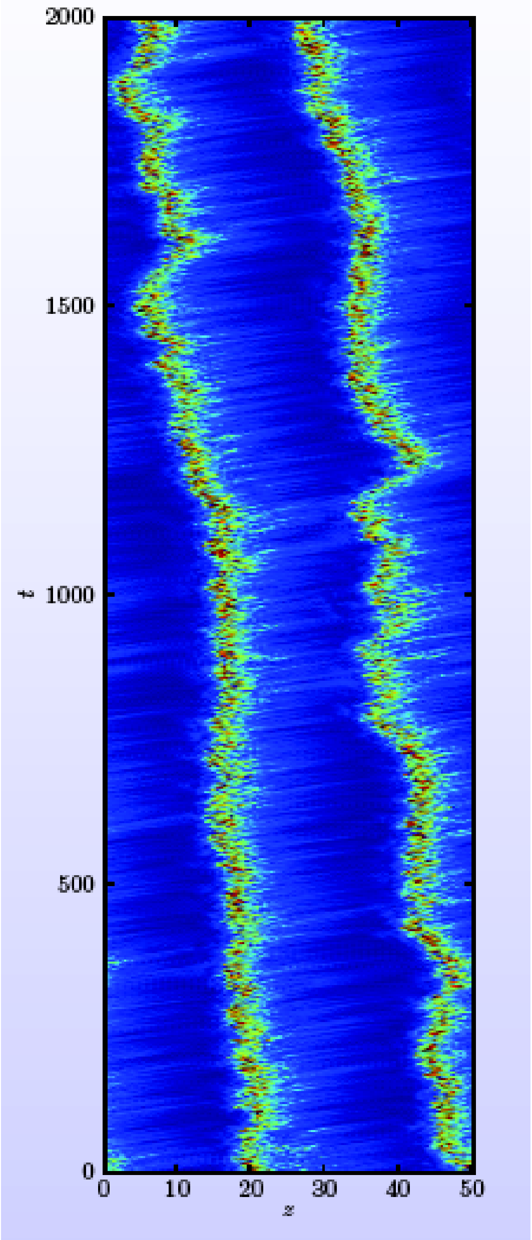
Cros & Le Gal (2002)

Plane Poiseuille



Tsukahara et al (2005)

Pipe Flow

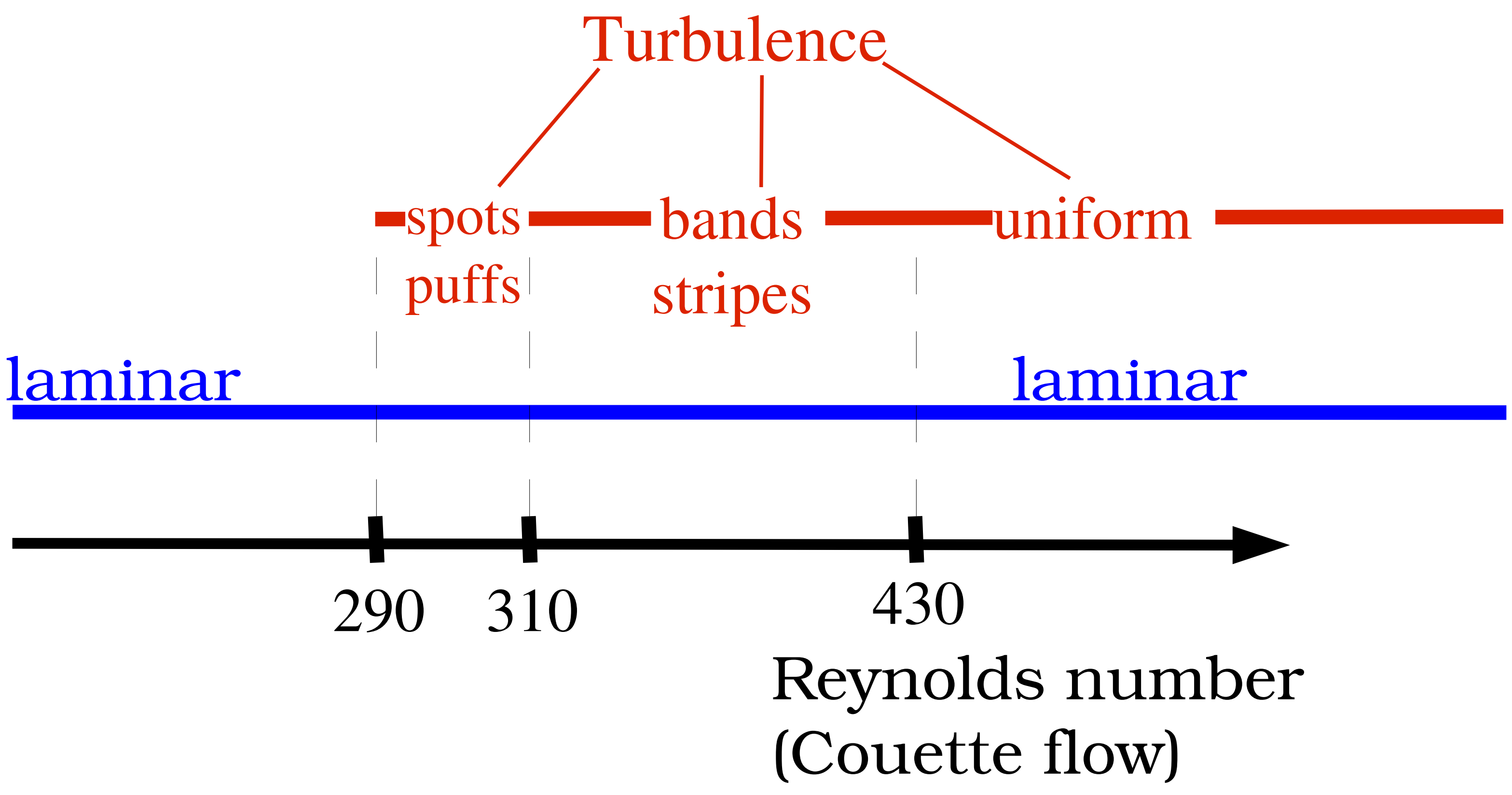


Moxey & Barkley (2009)

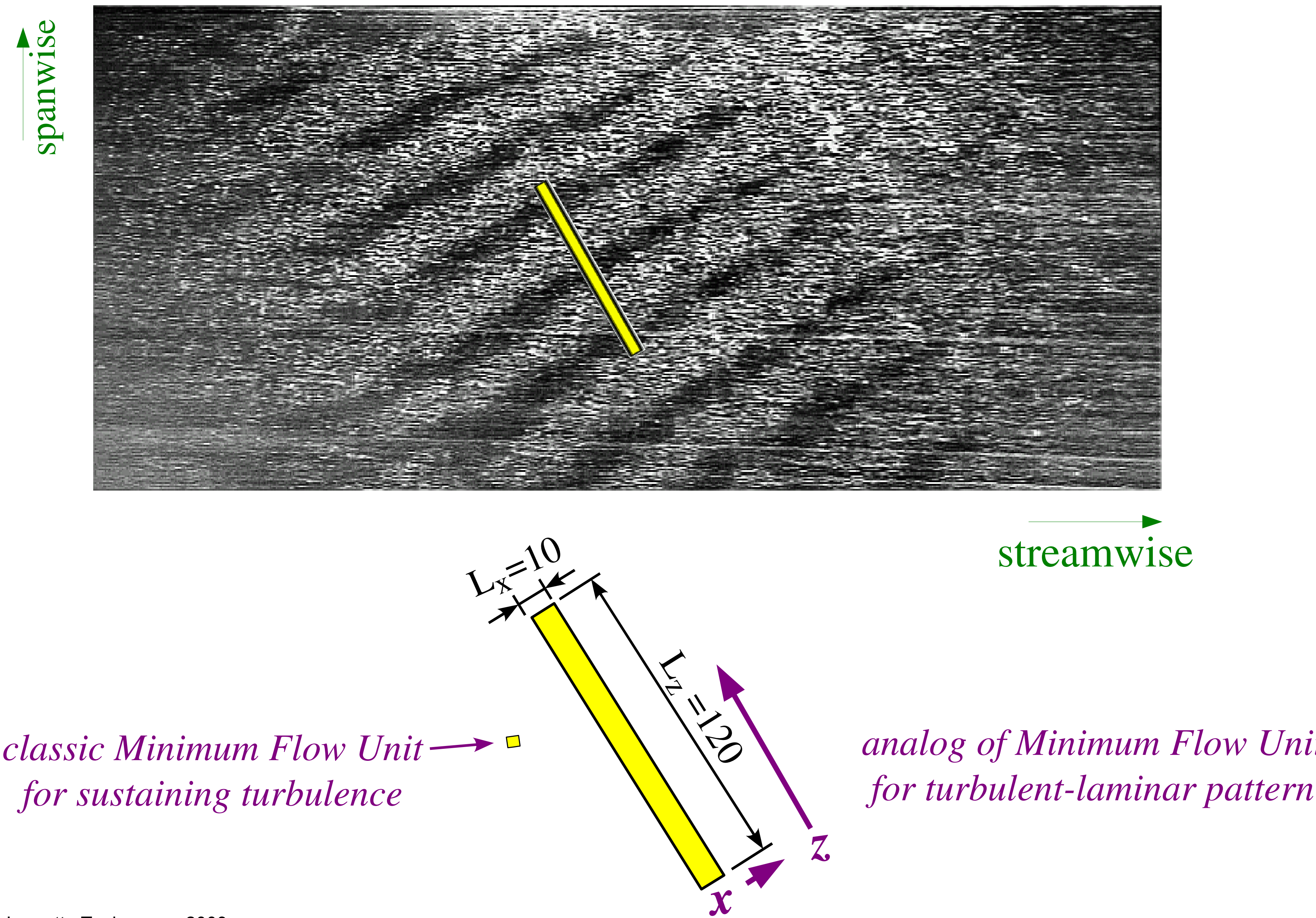
Plane Couette & Taylor-Couette:

- Manneville, Lagha, Rolland
- Duguet, Schlatter, Henningson
- Garcia-Villalba et al.
- Marques, Meseguer, Avila
- Dong

*In a **LARGE** box, turbulence takes **varied forms** near transition*
***bistable** with laminar flow*



Computational Domains: Angles and Size



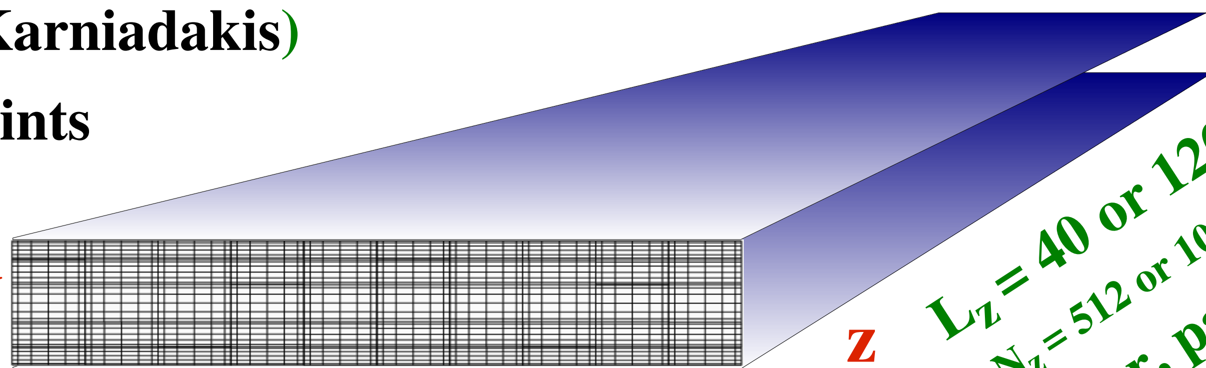
Numerical Methods

Direct Numerical Simulations of Navier-Stokes Equations

$$\partial_t \mathbf{u} = -(\mathbf{u}\cdot\nabla)\mathbf{u} + \frac{1}{Re}\Delta\mathbf{u} - \nabla p$$
$$\nabla\cdot\mathbf{u} = 0$$

`Prism` (Henderson & Karniadakis)

2 to 20 million gridpoints



Centre National de la Recherche Scientifique

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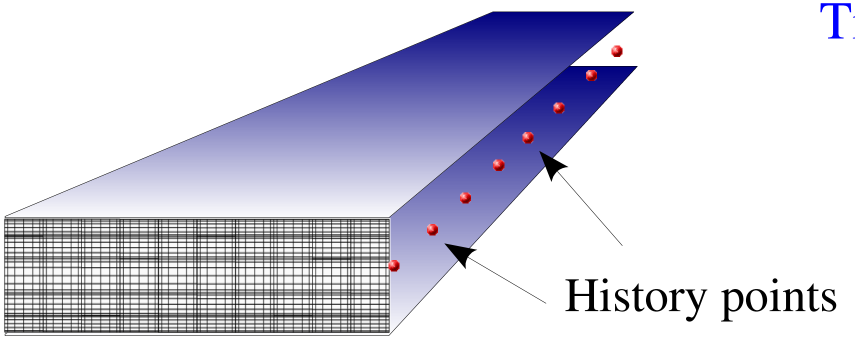
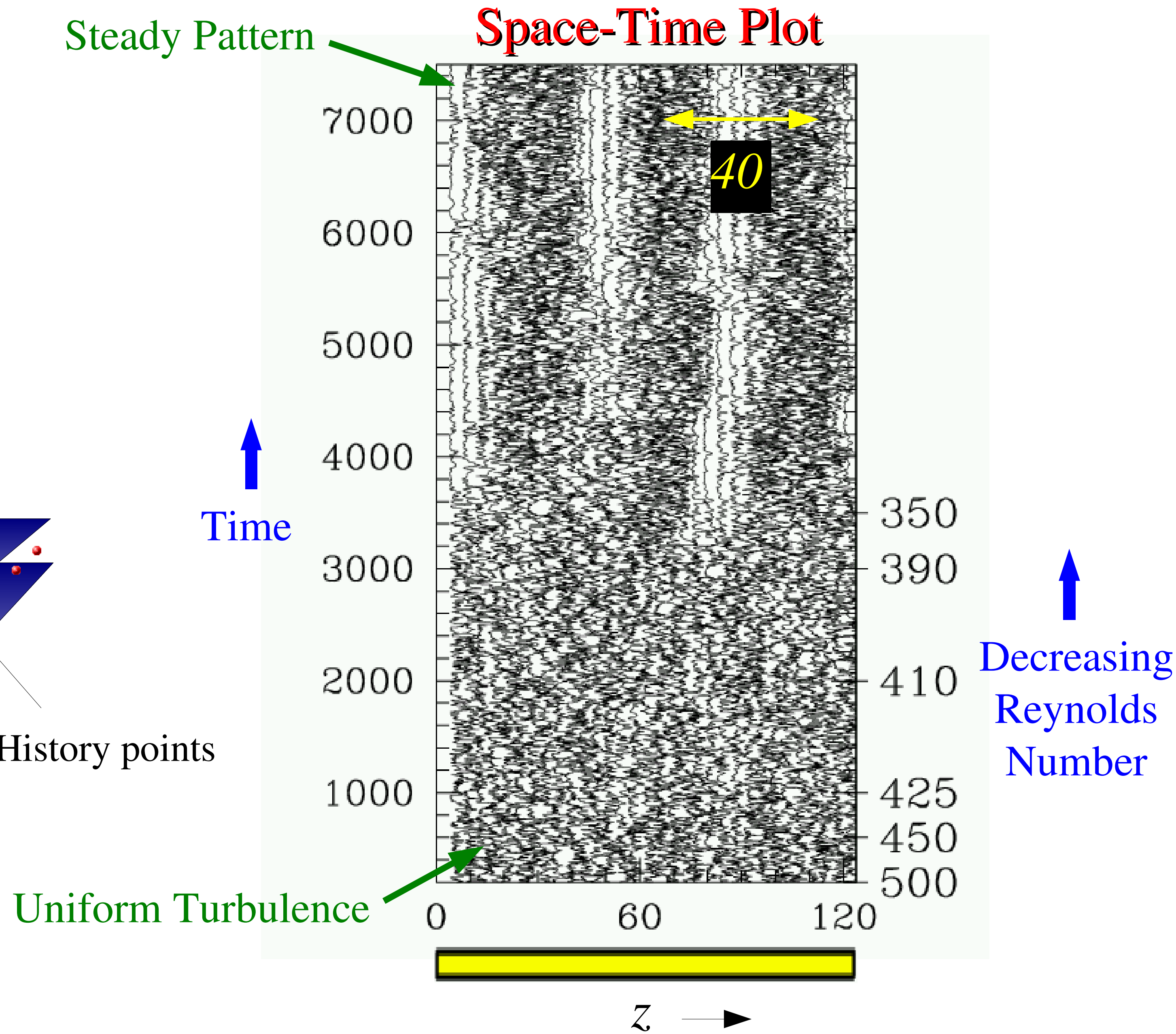
Results

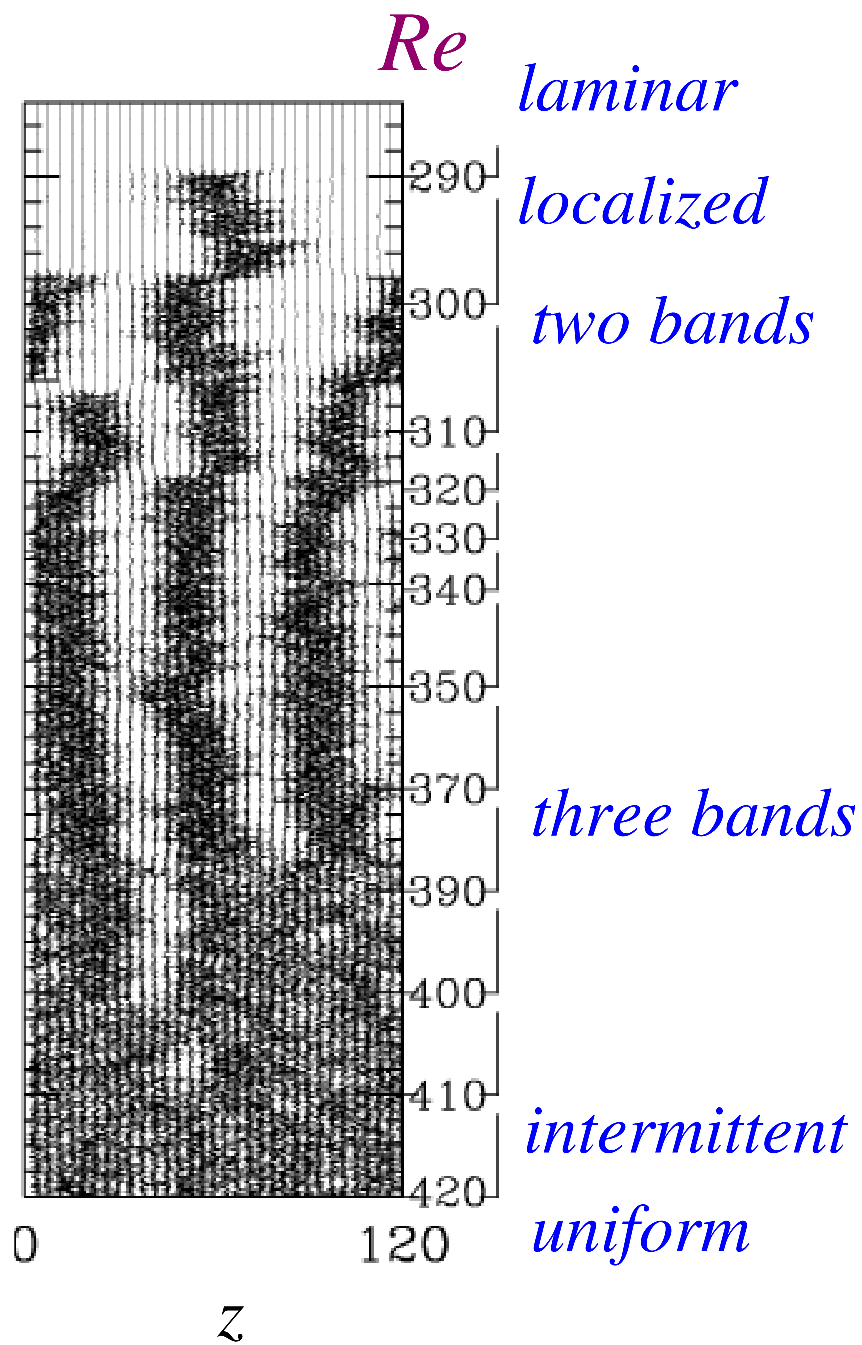
For each domain:

- Start at *Re = 500*
- Obtain turbulent flow
- Decrease *Re*
- Monitor turbulence



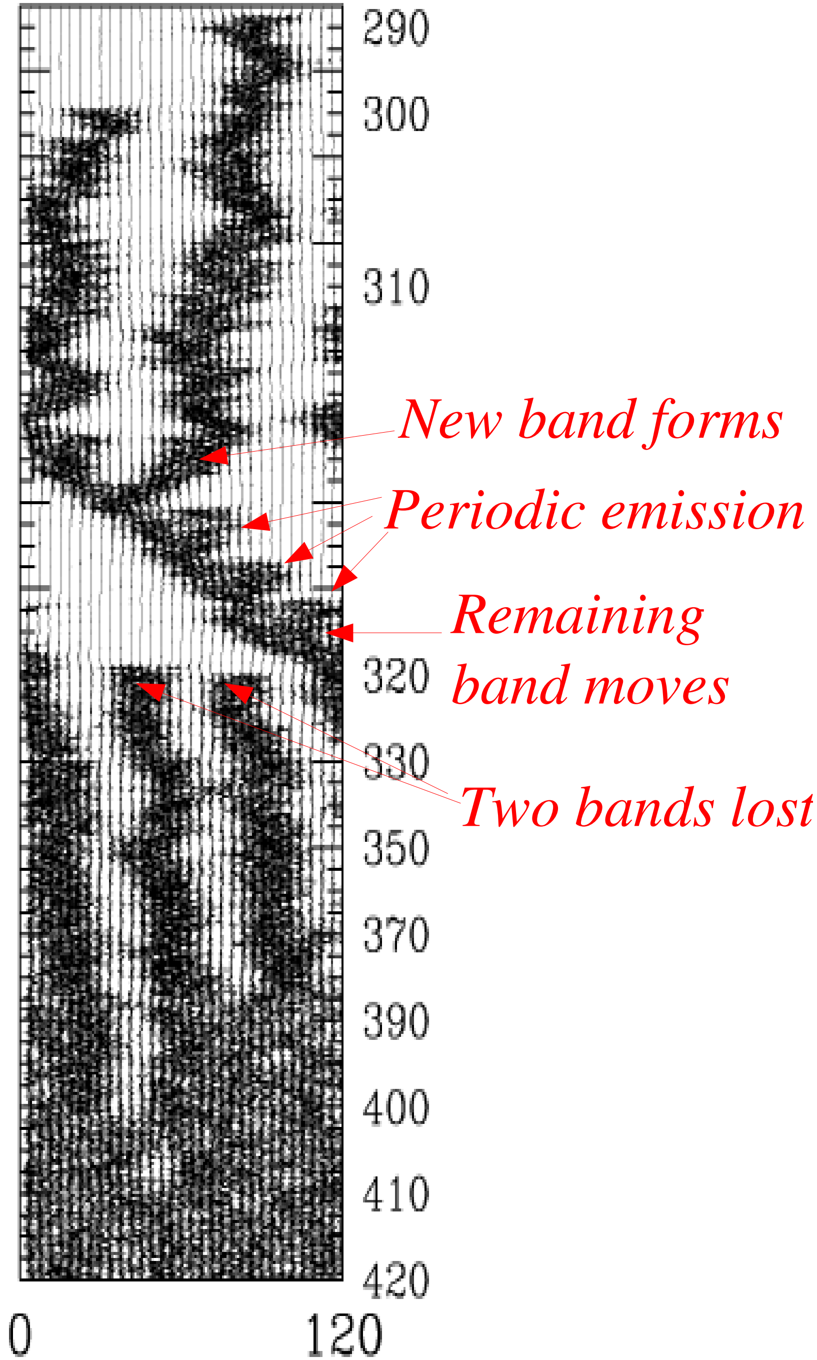


Six regimes



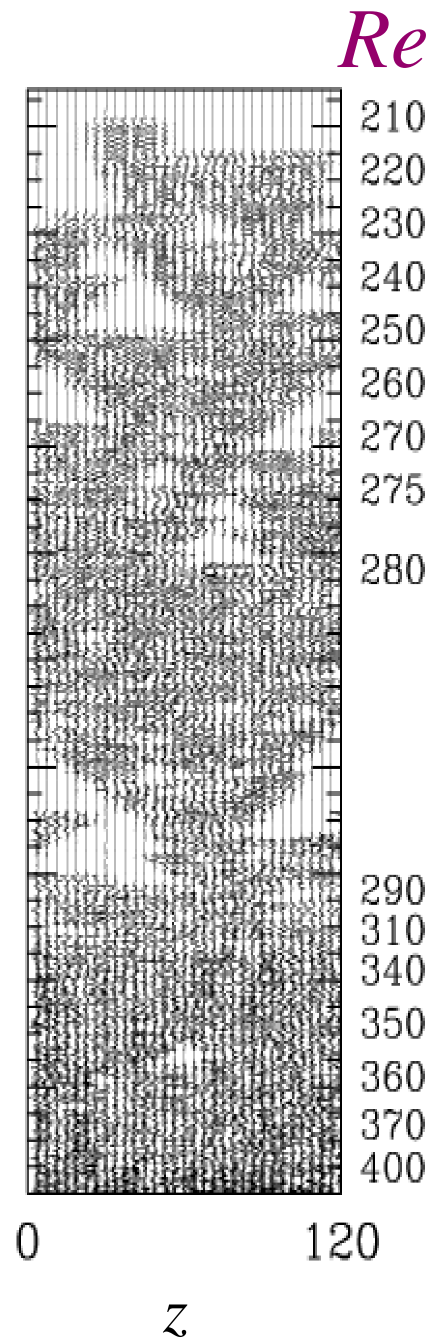
Re
laminar
localized
two bands
three bands
intermittent
uniform
290
300
310
320
330
340
350
370
390
400
410
420
0
120
z

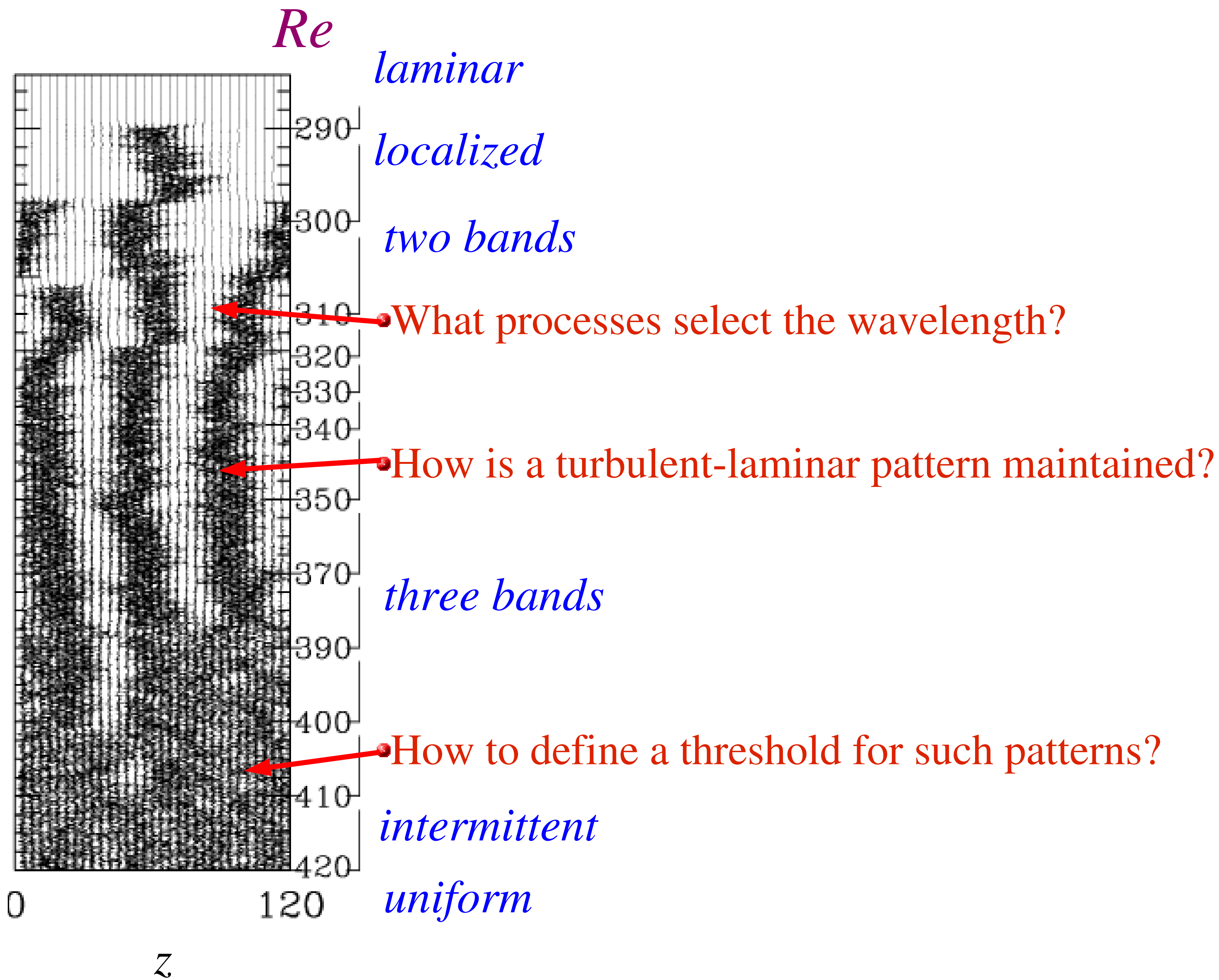
Branching (θ=24°)



STI (θ=0°)

long spanwise box

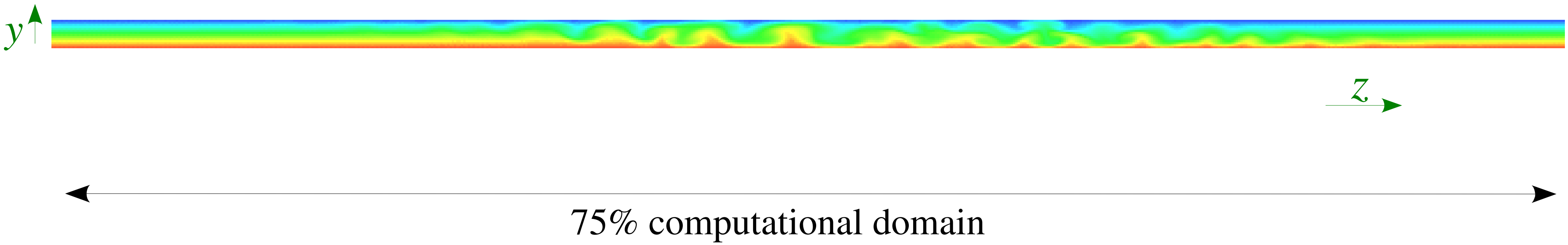



Re
laminar
290
localized
300
two bands
310
What processes select the wavelength?
320
330
340
How is a turbulent-laminar pattern maintained?
350
370
three bands
390
400
How to define a threshold for such patterns?
410
intermittent
420
uniform
0
120
z

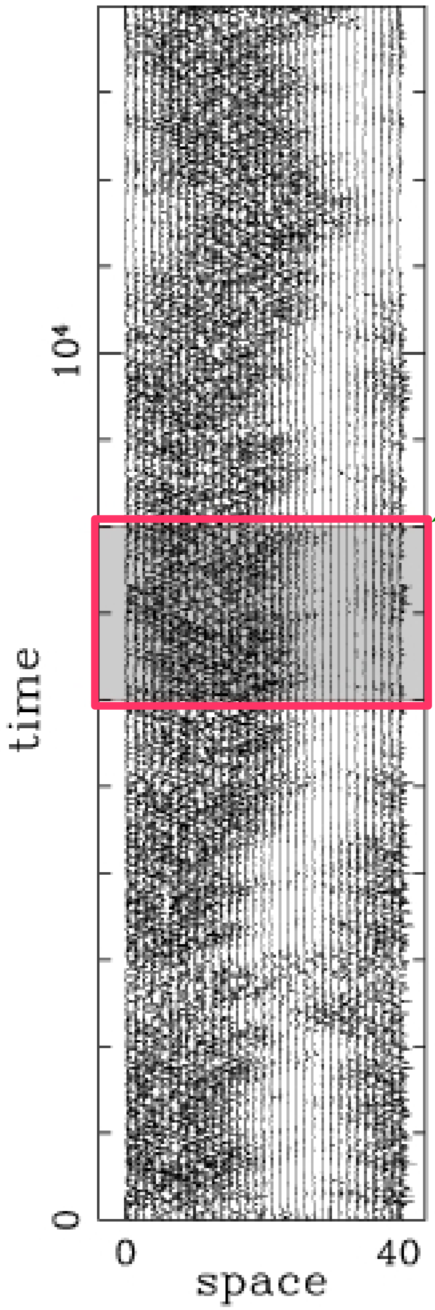
Movie of Localized State

Streamwise velocity in *y*-*z* plane

Re=300

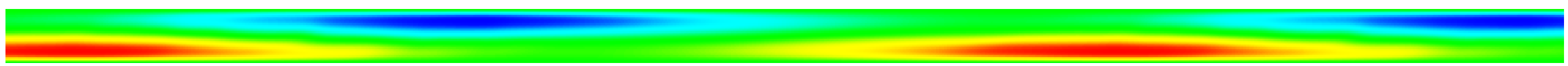


Average: in <u>time</u> and in <u>x</u> (short direction)



$$\mathbf{U}(y,z) \equiv \langle \mathbf{u} \rangle \equiv \int dx \int dt\, \mathbf{u}(x,y,z,t)$$

$$\tilde{\mathbf{u}} \equiv \mathbf{u} - \mathbf{U}$$



y

$U(y,z) - U_{\text{Couette}}$ $z \longrightarrow$

$\Psi(y,z) - \Psi_{\text{Couette}}$ (y-z streamfunction)

$\langle \tilde{\mathbf{u}} \cdot \tilde{\mathbf{u}} \rangle / 2$ turbulent kinetic energy

$u(x=0,y=0,z,t)$

$P(y,z)$ pressure

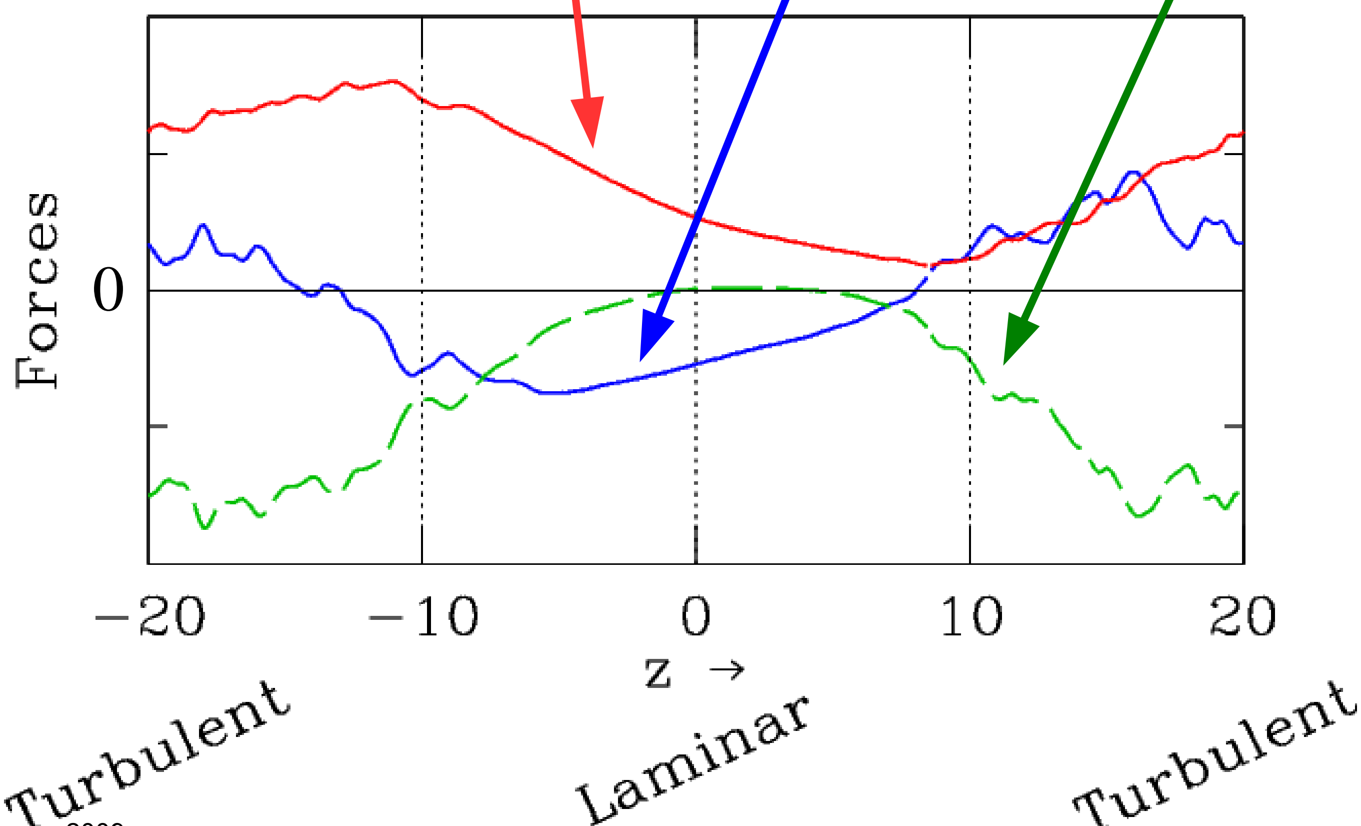
Balance of forces in U direction

$$\partial_t \mathbf{u} = -\nabla p + \frac{1}{Re}\Delta \mathbf{u} - (\mathbf{u}\cdot\nabla)\,\mathbf{u}$$

$$0 = -\nabla P + \frac{1}{Re}\Delta \mathbf{U} - (\mathbf{U}\cdot\nabla)\,\mathbf{U} - \langle(\tilde{\mathbf{u}}\cdot\nabla)\,\tilde{\mathbf{u}}\rangle$$

average

Viscous — *Nonlinear* — *Reynolds stress*



Wavelength Selection

$$\mathbf{U}_{\text{Couette}} = (\hat{\mathbf{e}}_x \cos\theta + \hat{\mathbf{e}}_z \sin\theta) y$$

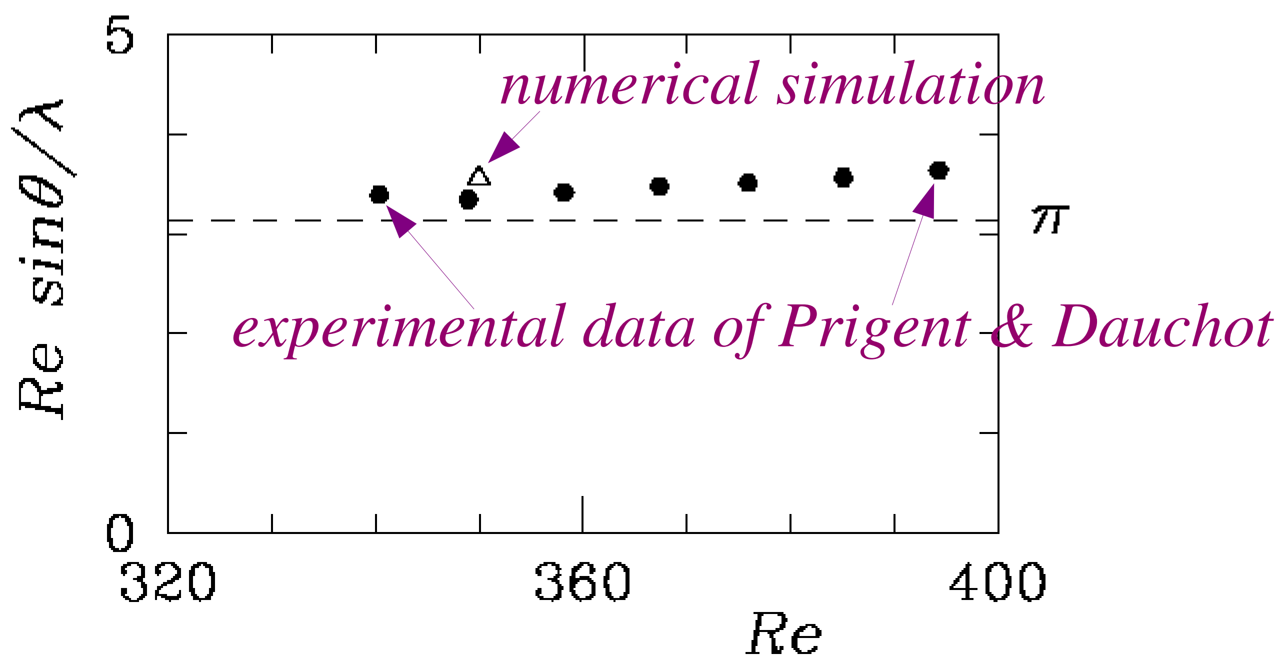
$$\boxed{\sin\theta \; y \; \partial_z U} = \boxed{\frac{1}{Re} \, \partial_y^2 U}$$

$$\partial_y^2 \gg \partial_z^2$$

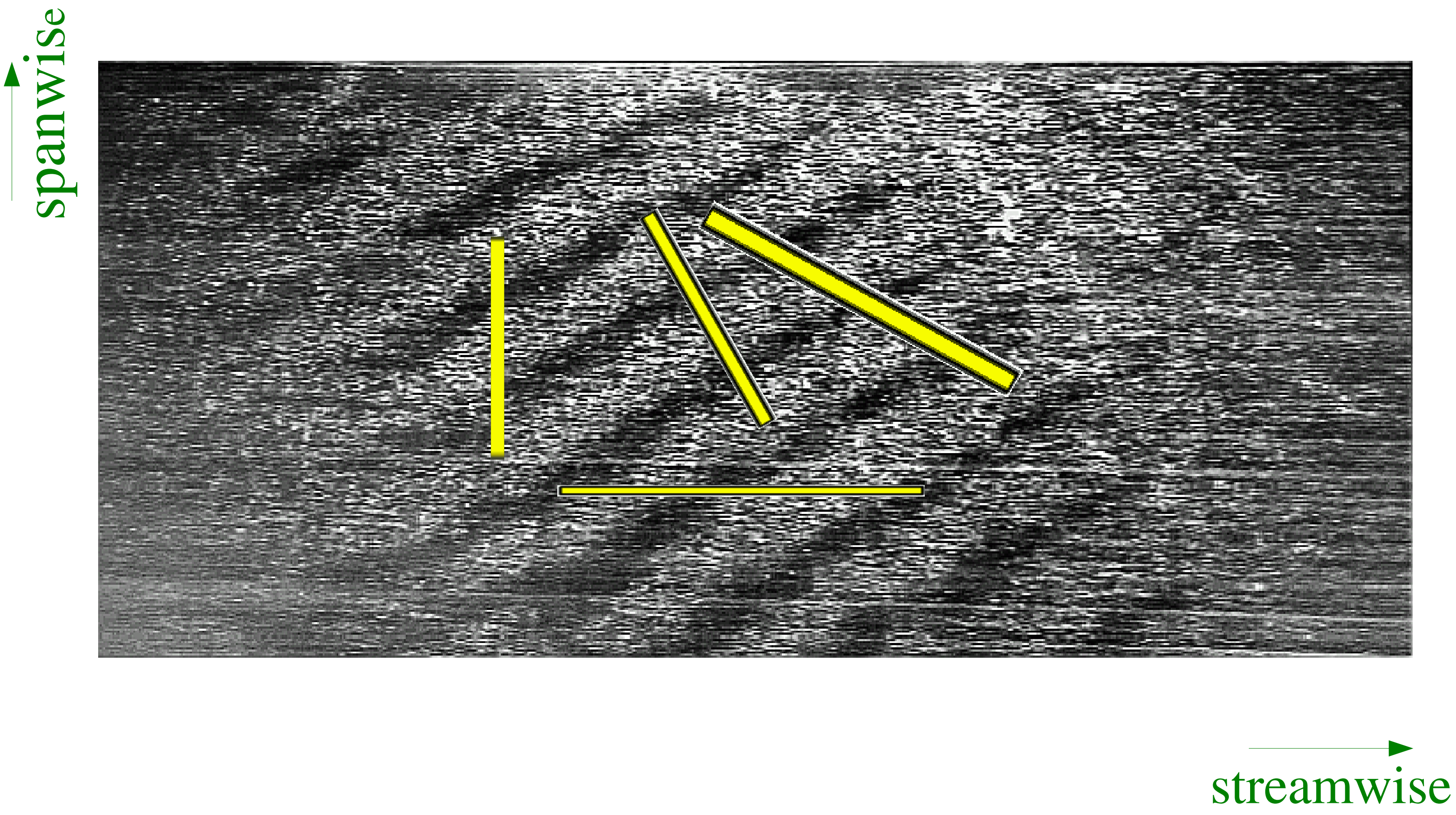
(boundary layer theory)

$$\sin\theta \; \frac{1}{2} \; \frac{2\pi}{\lambda} \approx \frac{1}{Re} \, \pi^2$$

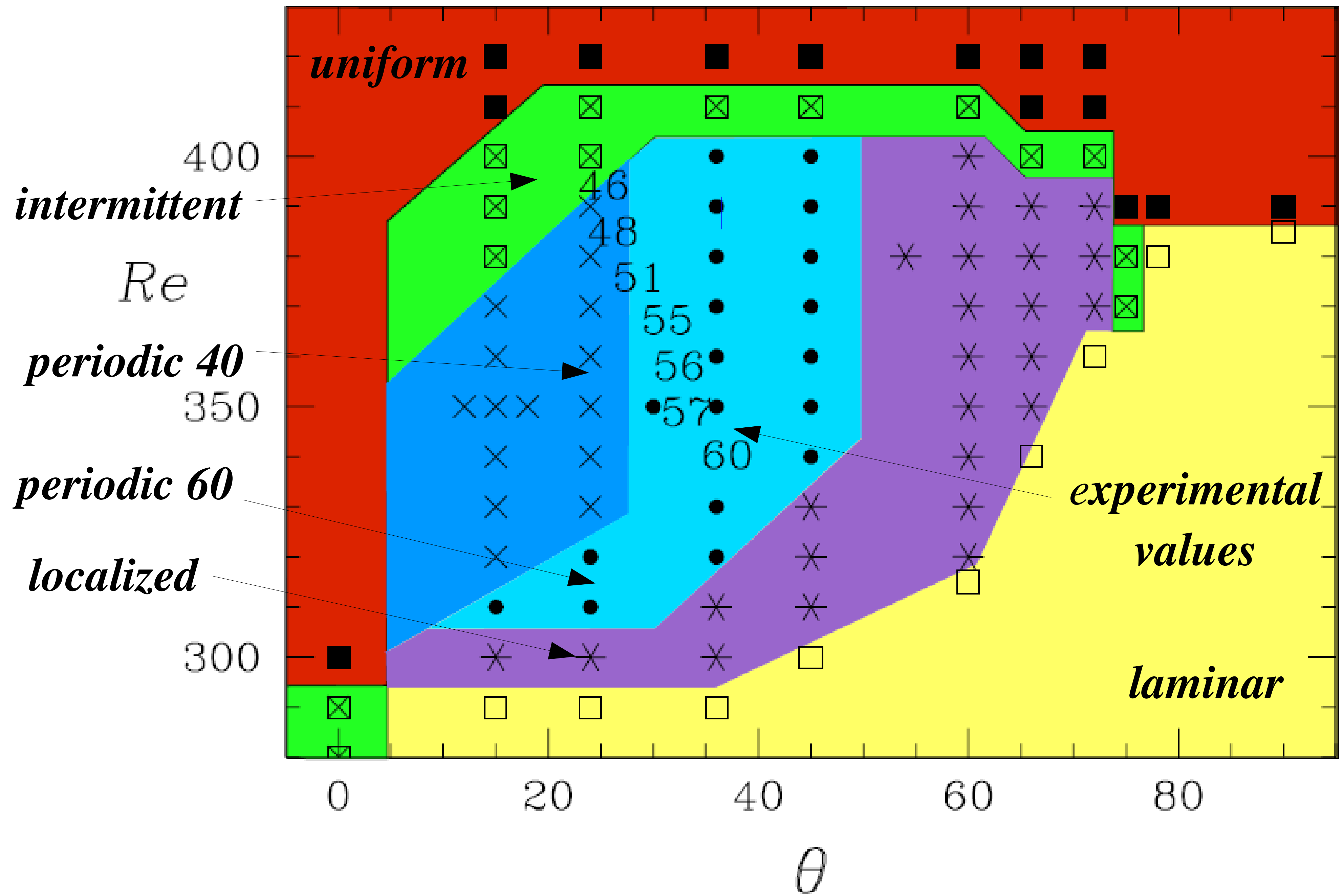
$$\frac{Re \; \sin\theta}{\lambda} \approx \pi$$



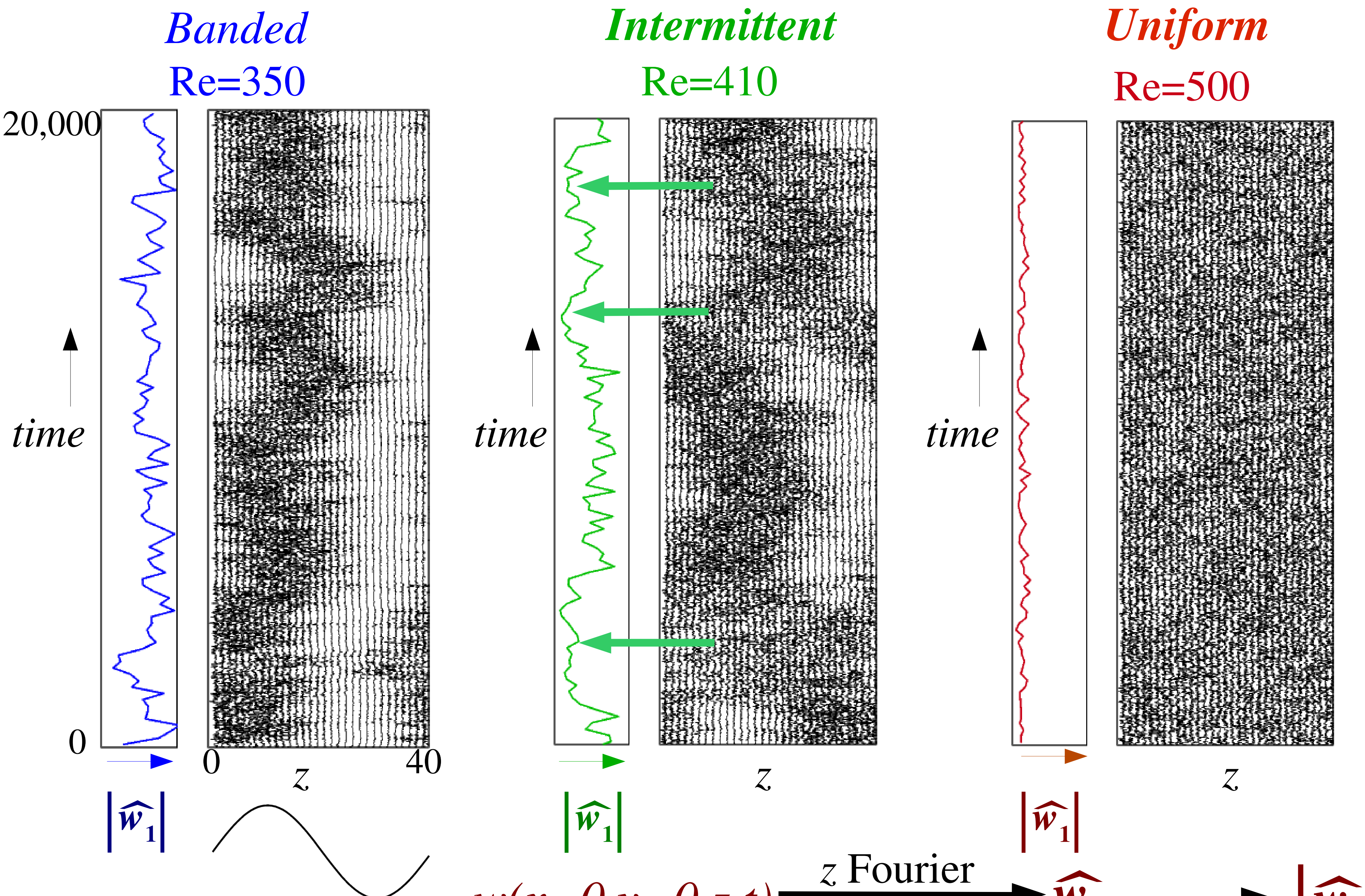
Computational Domains: Angles and Size



Varying angle: Regimes as a function of θ, Re

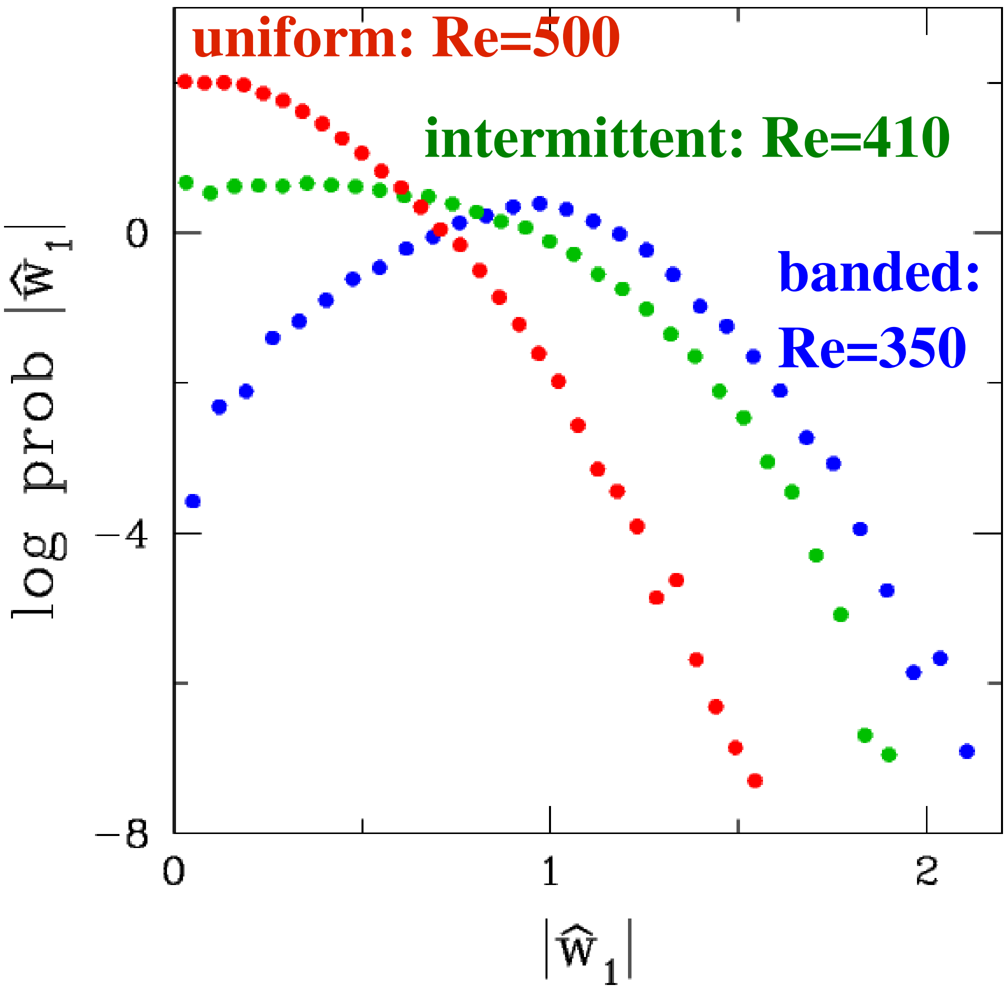


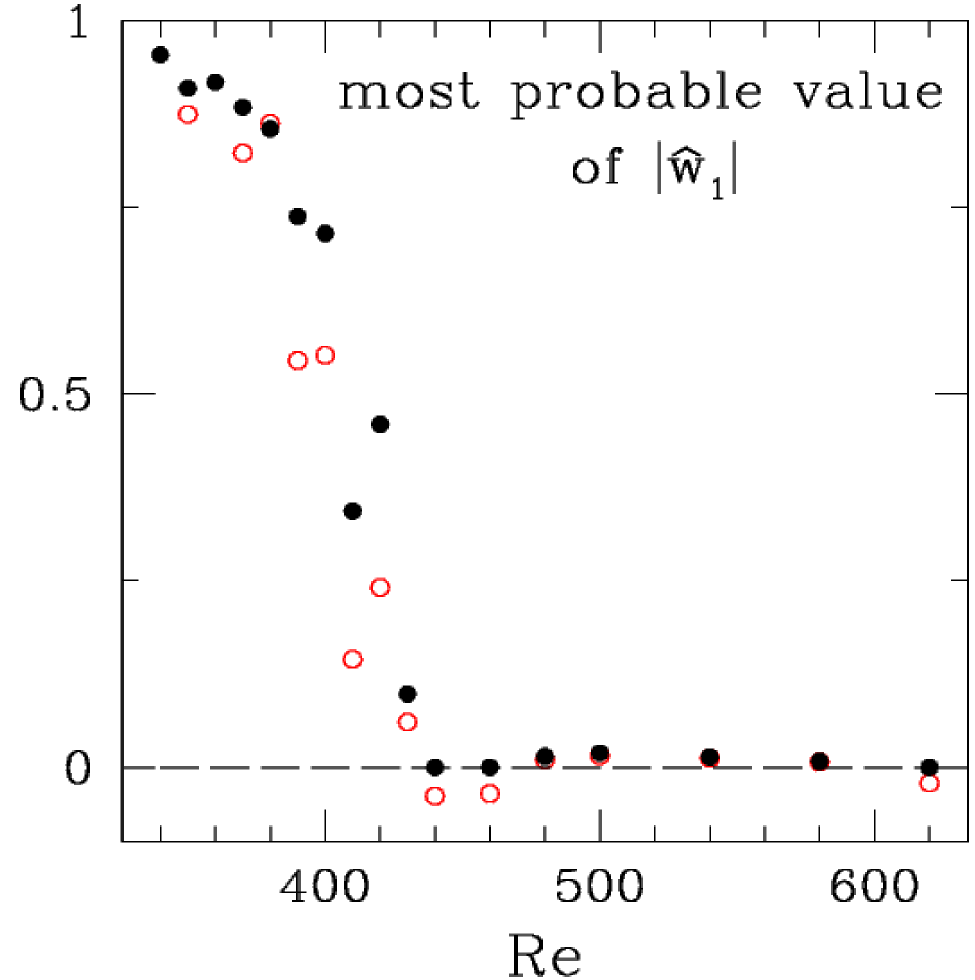
Turbulent Timeseries



$$w(x=0,y=0,z,t) \xrightarrow[\text{transform}]{z\ \text{Fourier}} \hat{w}_1 \longrightarrow |\hat{w}_1|$$

Probability Distribution Function of $|\hat{w}_1|$
(modulus of m=1, λ=40 component of spanwise velocity)





Conclusions

❇ Can reproduce experimental turbulent-laminar pattern in a tilted minimal domain

❇ Average over x and t yields mean flow $\boldsymbol{U}(y,z)$ which satisfies non-trivial balance between viscous and nonlinear terms in quasi-laminar region (not linear in y)
Leads to relation between *Re,* tilt angle θ and wavelength λ

❇ Most probable value of spatial Fourier coefficient is a good order parameter for the transition to turbulent-laminar patterns.