Artificial gauge fields and Zitterbewegung in a BEC

I. B. Spielman

Current team

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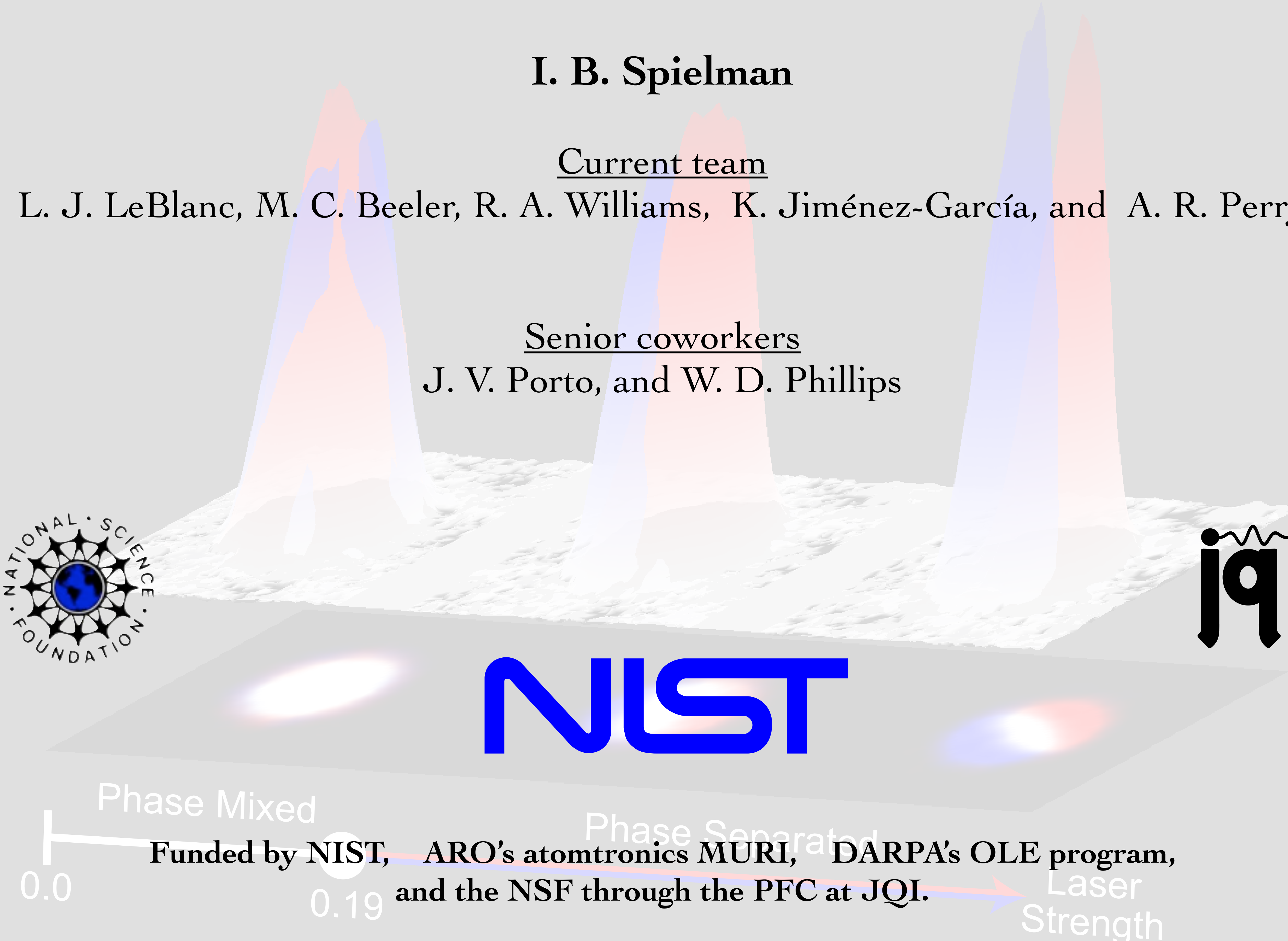
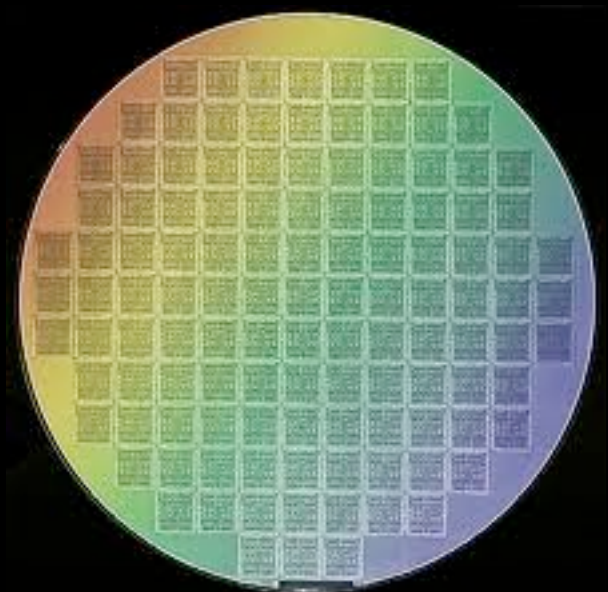
Senior coworkers

J. V. Porto, and W. D. Phillips

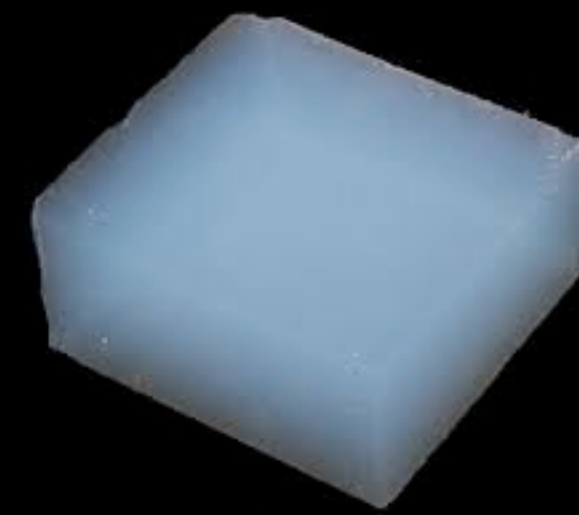


Funded by NIST, ARO's atomtronics MURI, DARPA's OLE program, and the NSF through the PFC at JQI.

What are materials?



Si
2.3 g/cm^3



Aerogel
1 mg/cm^3

Ian's answer: "chunks of stuff."

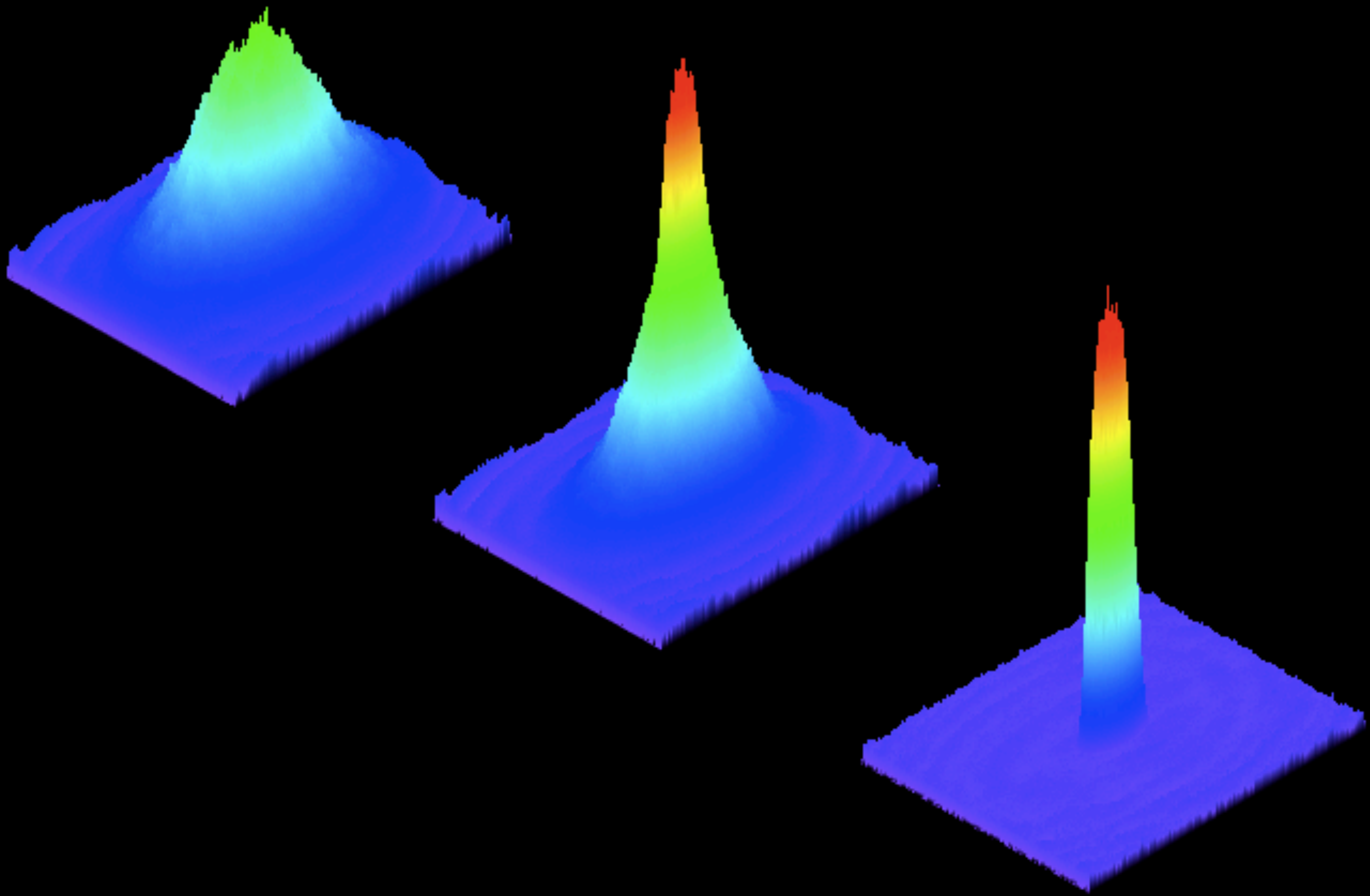
Liquid Helium
125 mg/cm^3

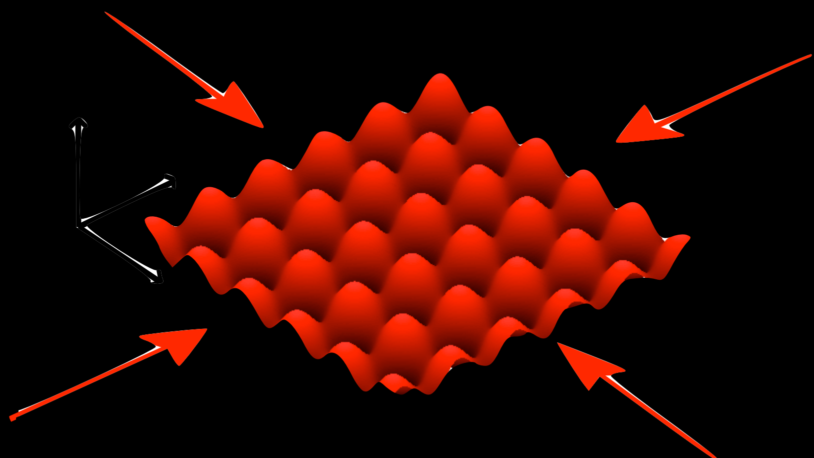
Ultracold neutral atoms

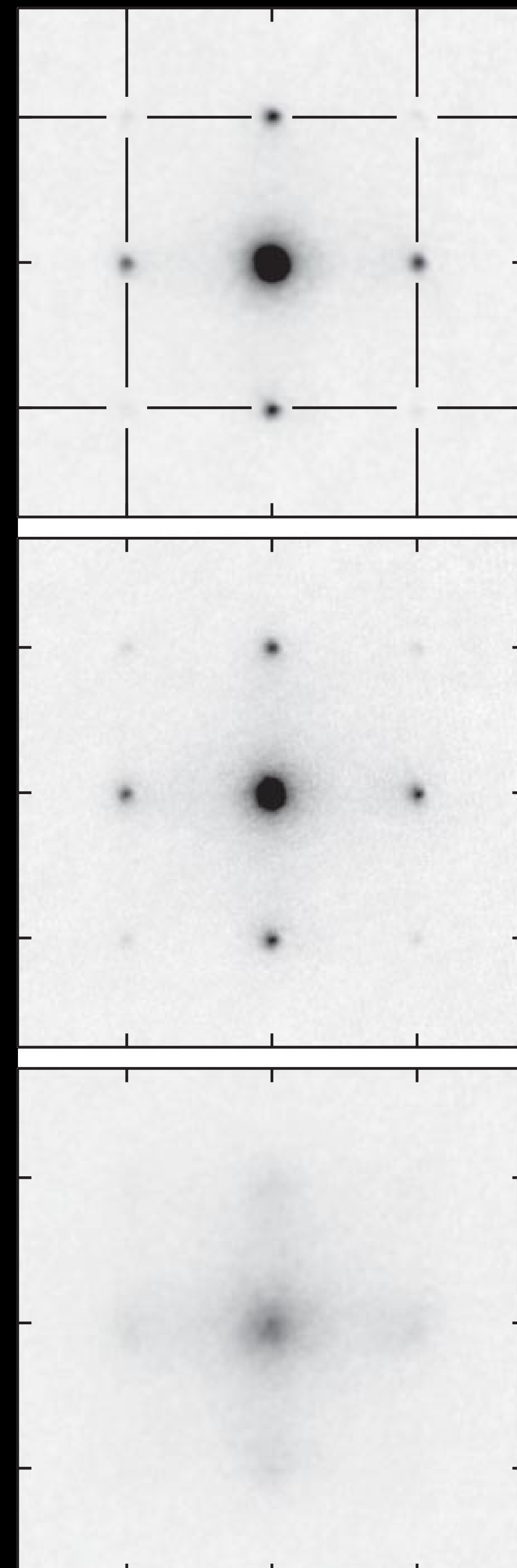
~10^{14} cm^{-3} or 100 ng/cm^3

(air is ~1 mg/cm^3)

Are these materials?







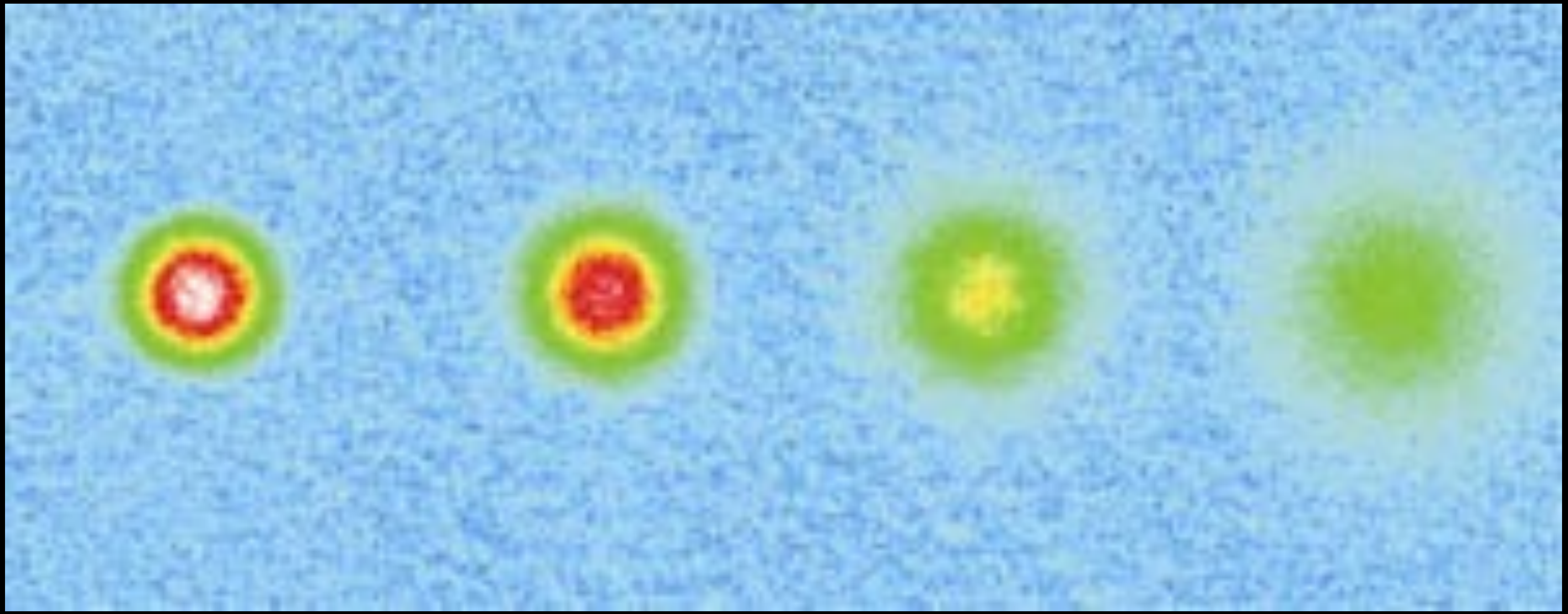
They can be fluids

They can be insulators

first: Greiner et al Nature (2002)

They can be bosons

They can be fermions

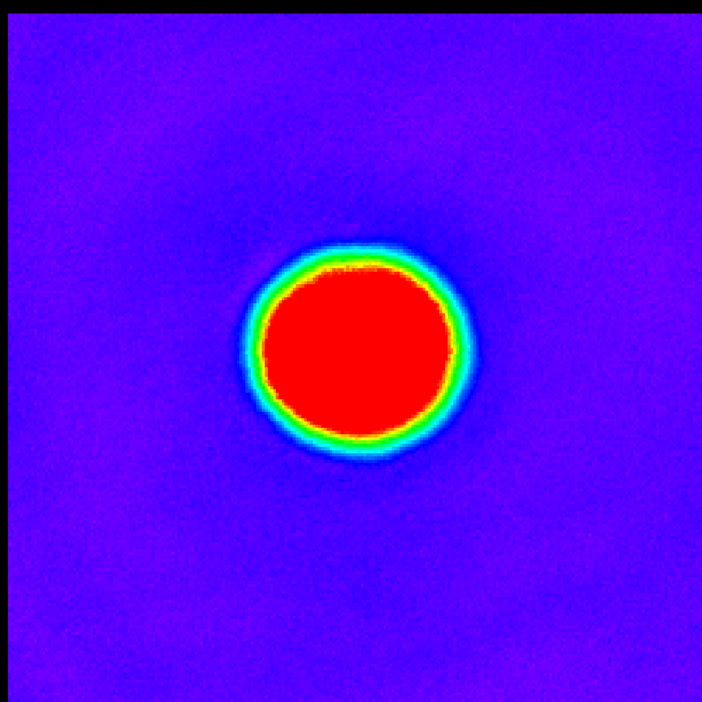


They can be molecules

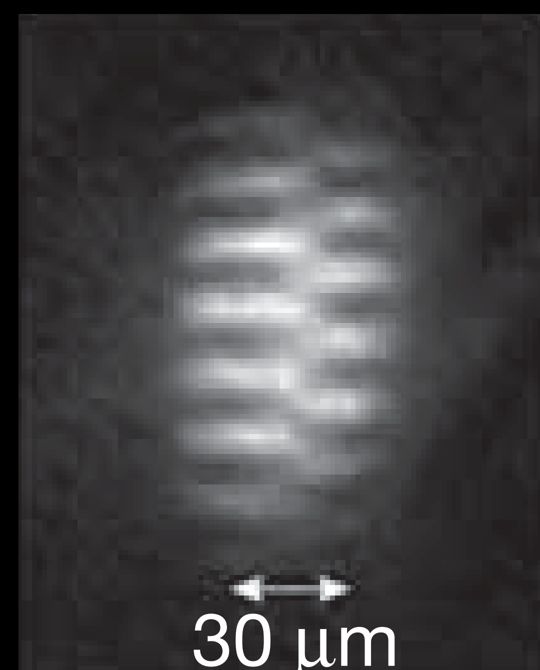
They can be atoms

e.g., Regal Nature (2003)

They can be 3D

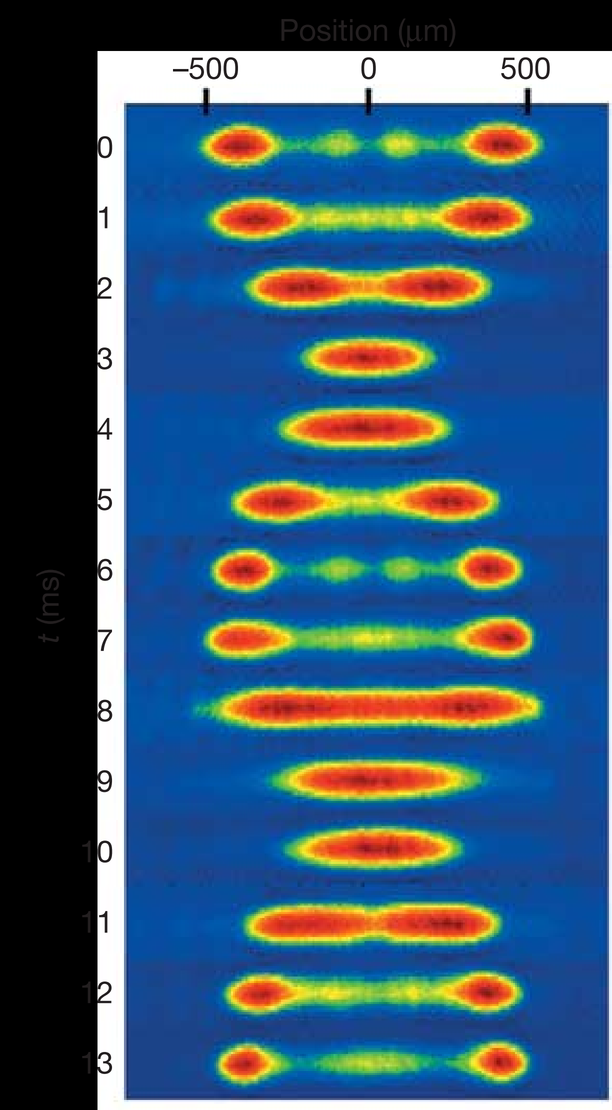


They can be 2D



e.g., Hadzibabic Nature (2006)

They can be 1D



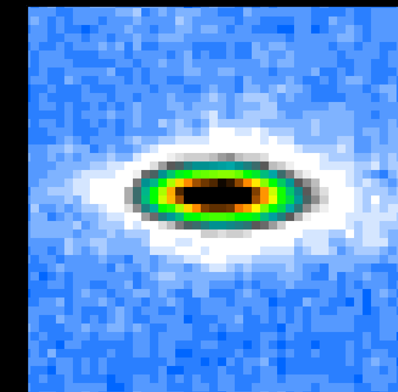
e.g., Kinoshita Nature (2006)

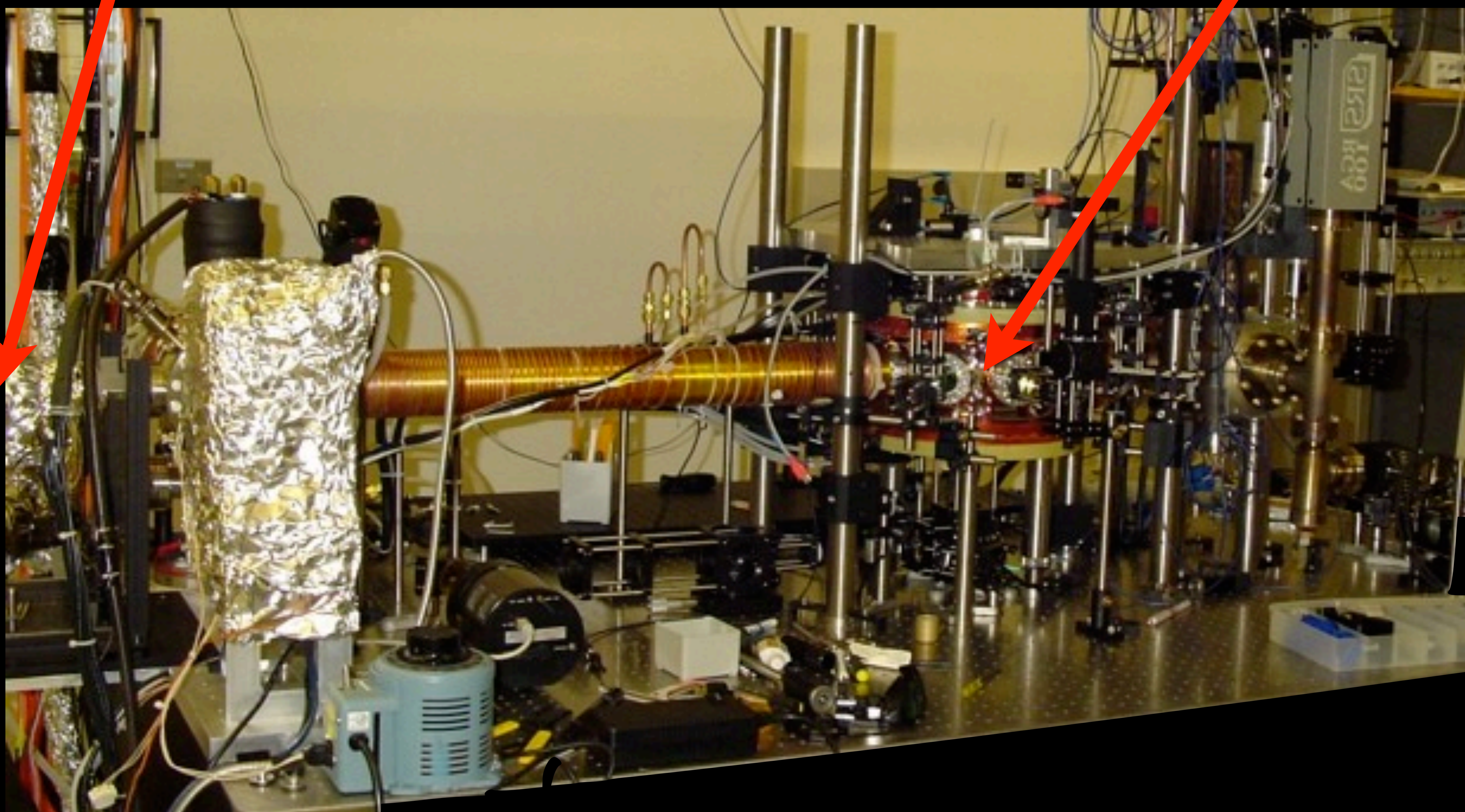
Starts like this

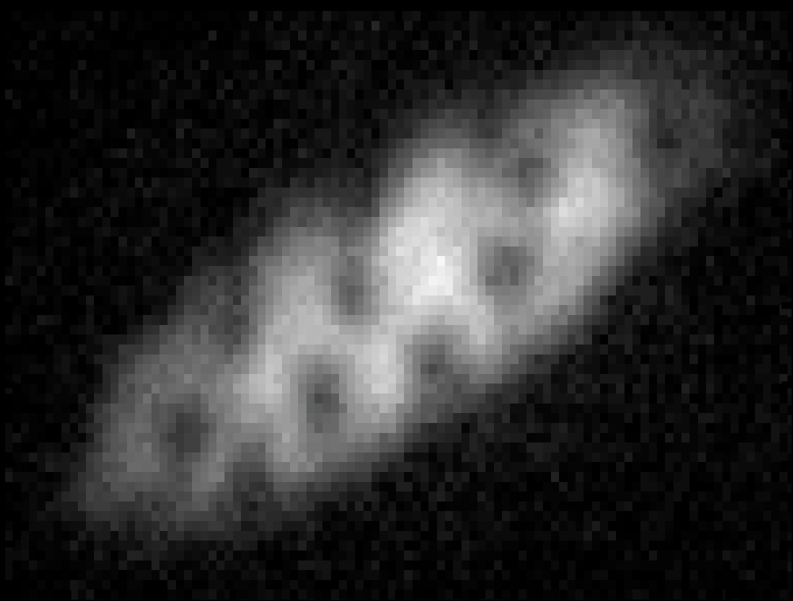
400 K

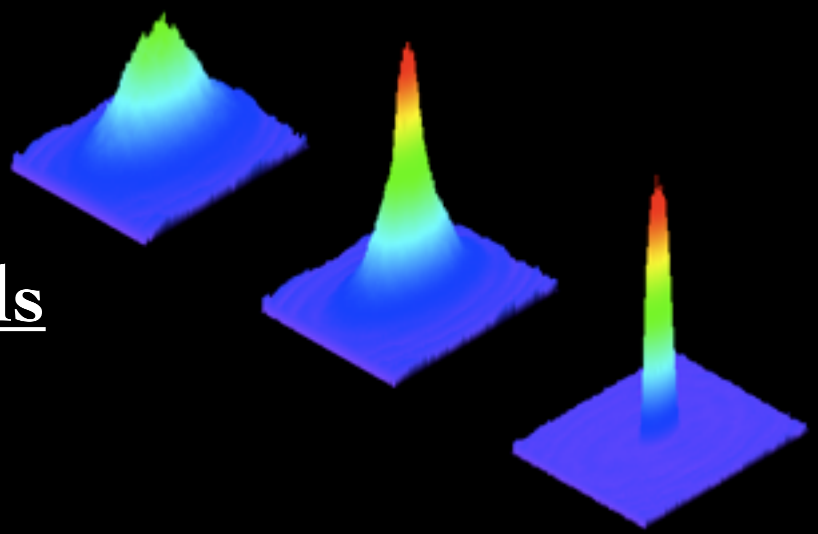
Ends here every 20 s

50 nK





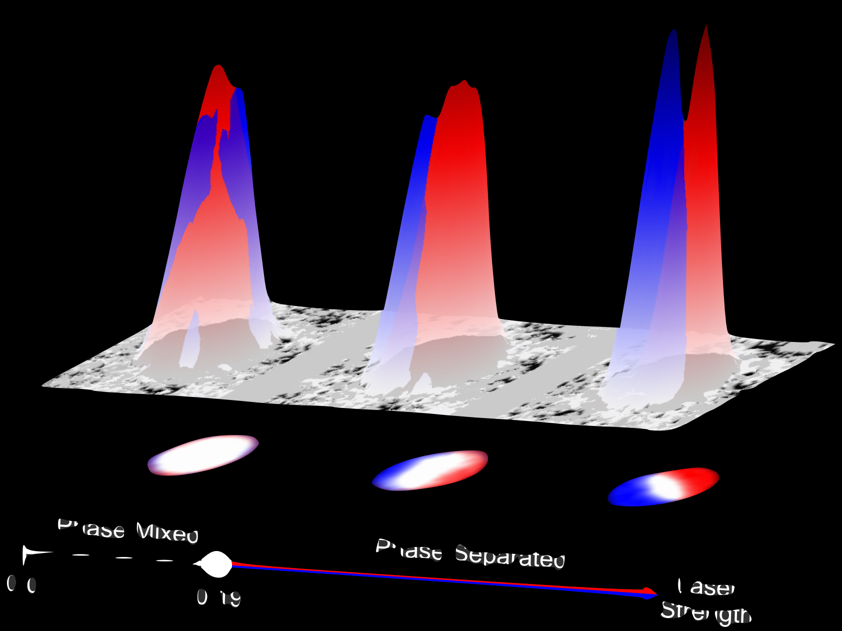




<u>Cold atoms are good materials</u>

Numerous properties can
be controlled and measured
on all relevant timescales and
in any lab

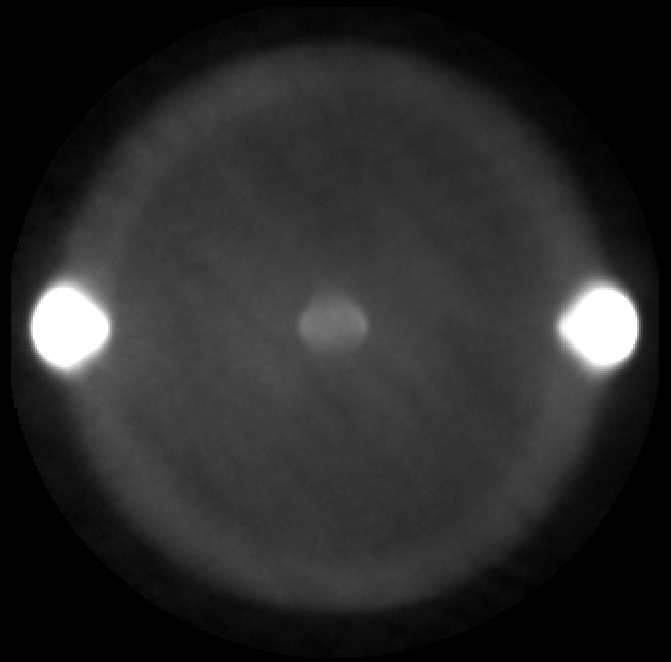
Very simple Hamiltonians



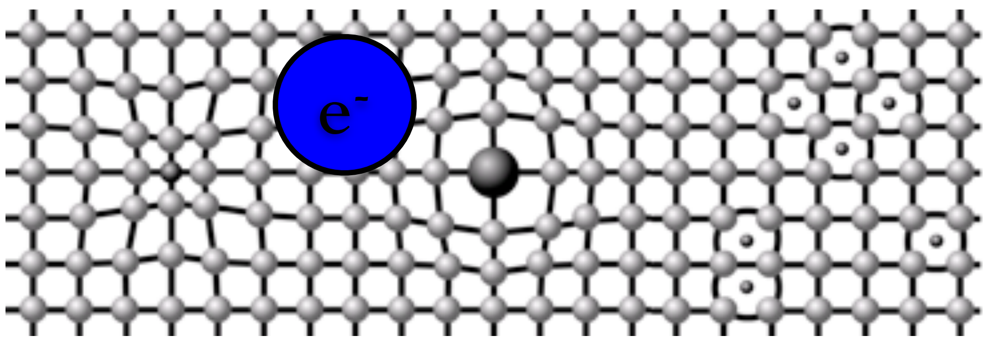
<u>Cold atoms are bad materials</u>

Short lived, and do so in vacuum

Interesting features **all** added
by hand (complex experiments).



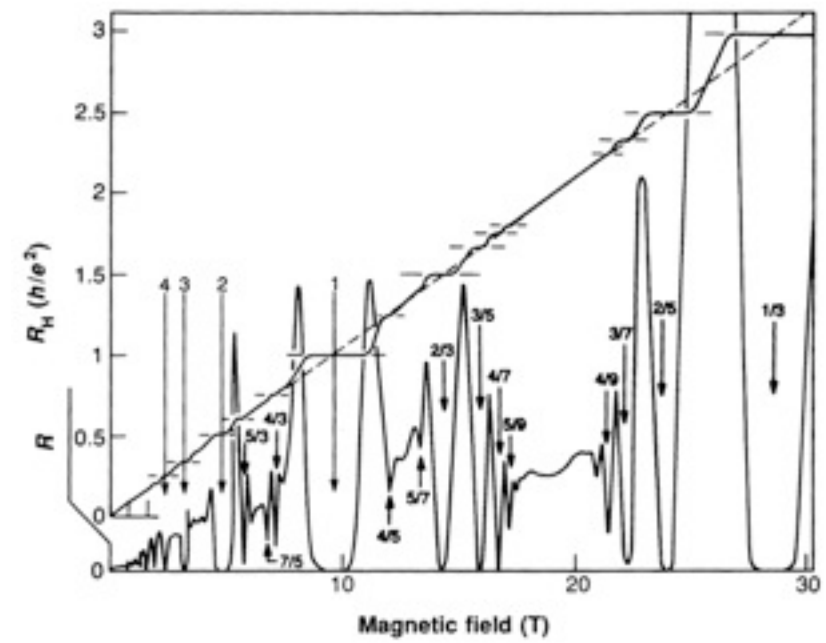
Electron



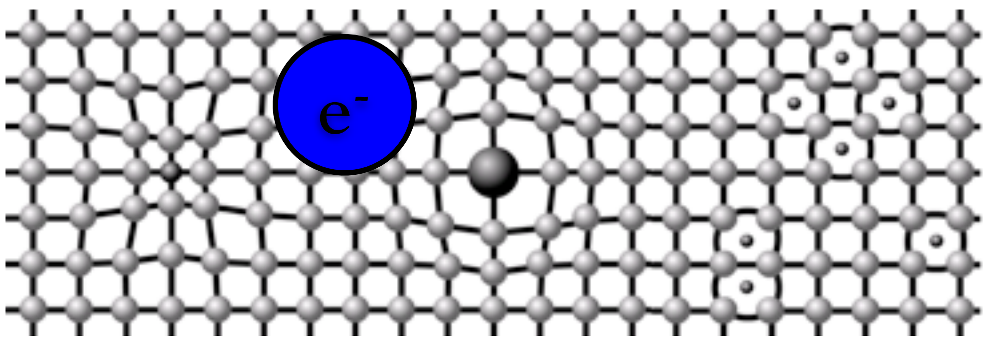
ab initio
understand is hard

Electrons

Materials



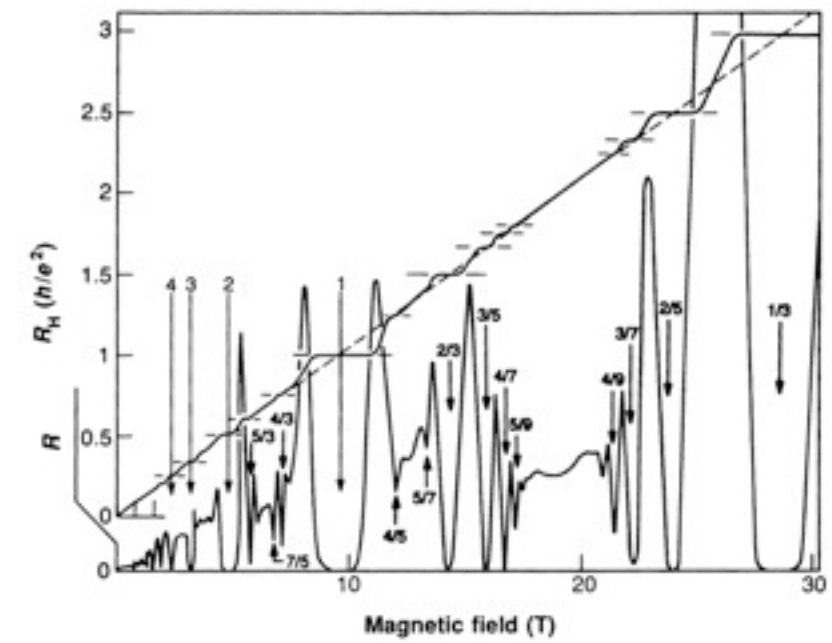
Electron



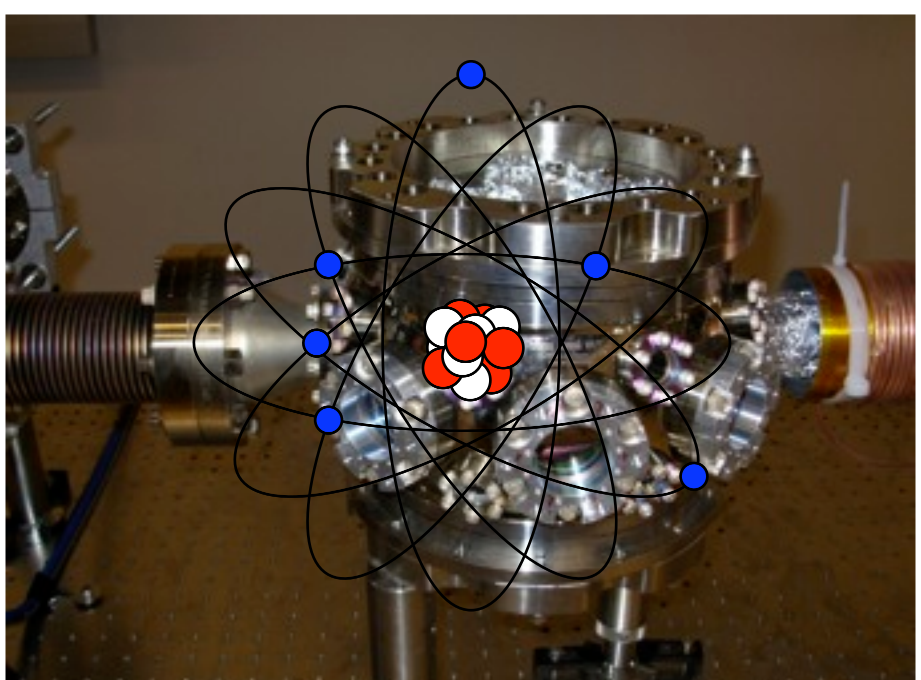
ab initio
understand is hard

Electrons

Materials



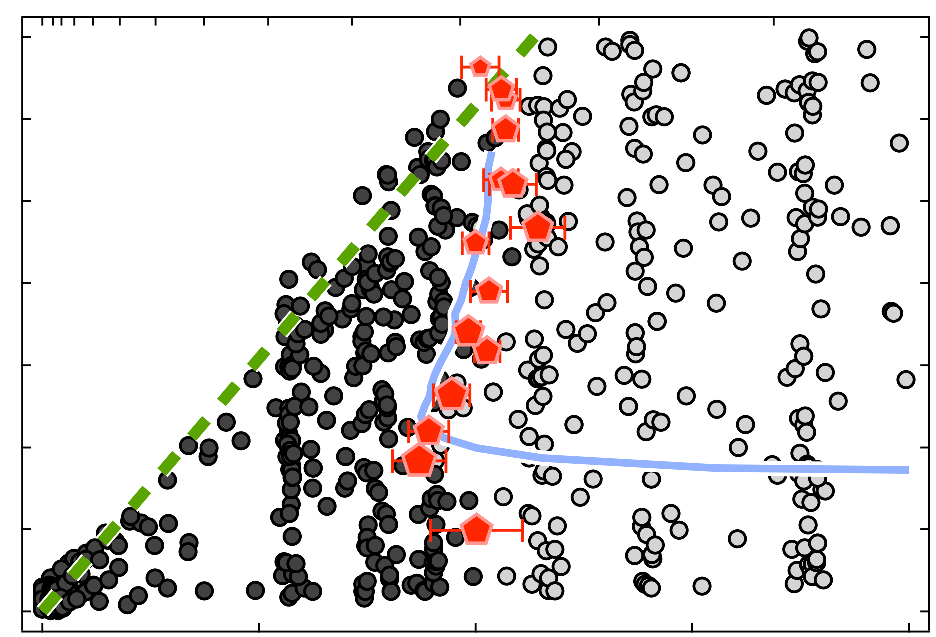
Atom



ab initio
is easier
(but still hard)

Atoms

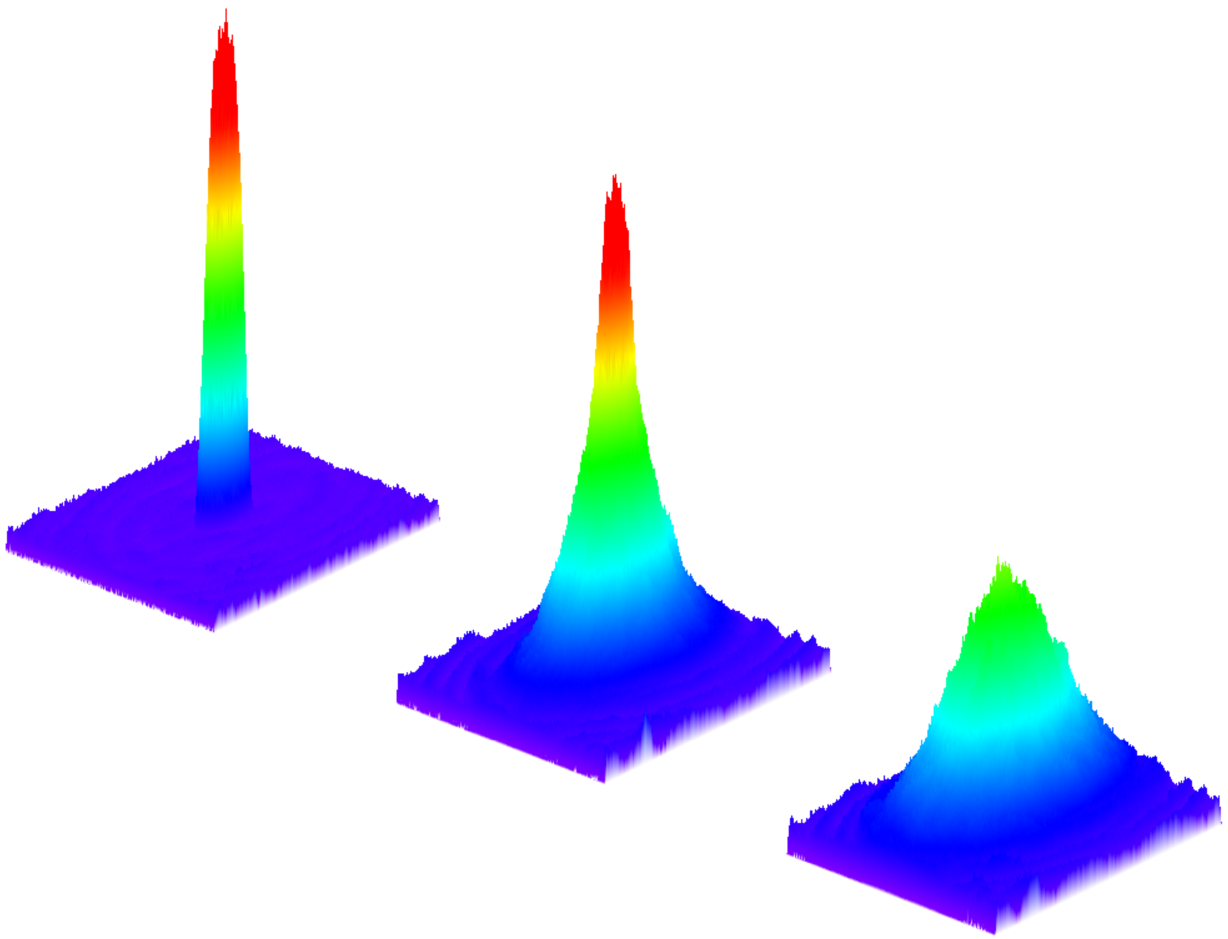
Atomic quantum materials

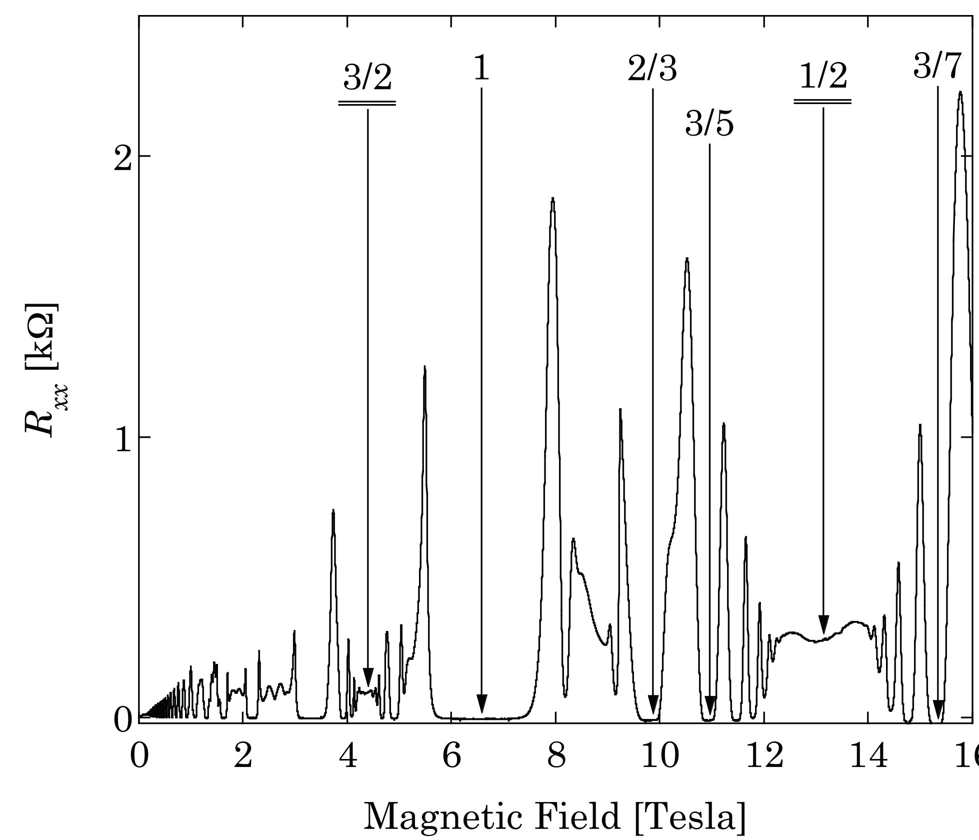


Vision: atoms + fields

Bose-Einstein condensation

Quantum Hall effects





Charge neutral bosons

Macroscopically occupied quantum state

<u>Weak interactions</u>

Phonon-like excitations, bosons

Charged fermions in a large magnetic field

Macroscopically degenerate ground state

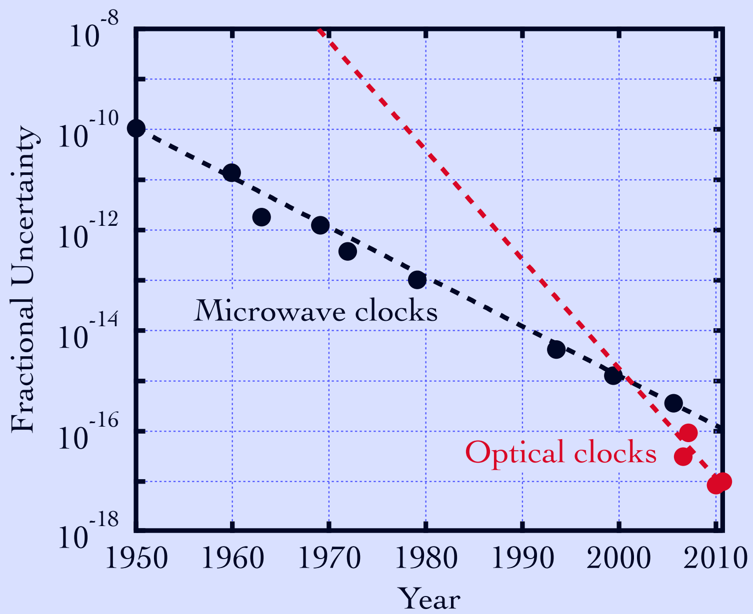
<u>Strong interactions</u>

Particle-like excitations with fractional quantum numbers

A brief history of atomic physics

Reduction

Very good understanding an atom



Assemble new systems

Make new quantum systems of **Atoms**

Control of external states

Many-body physics

Quantum simulation

Origin of complexity from simple components

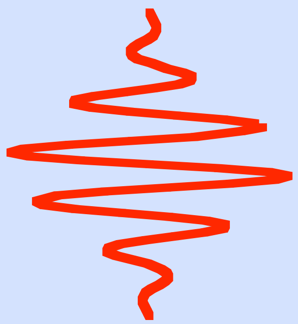
Quantum information

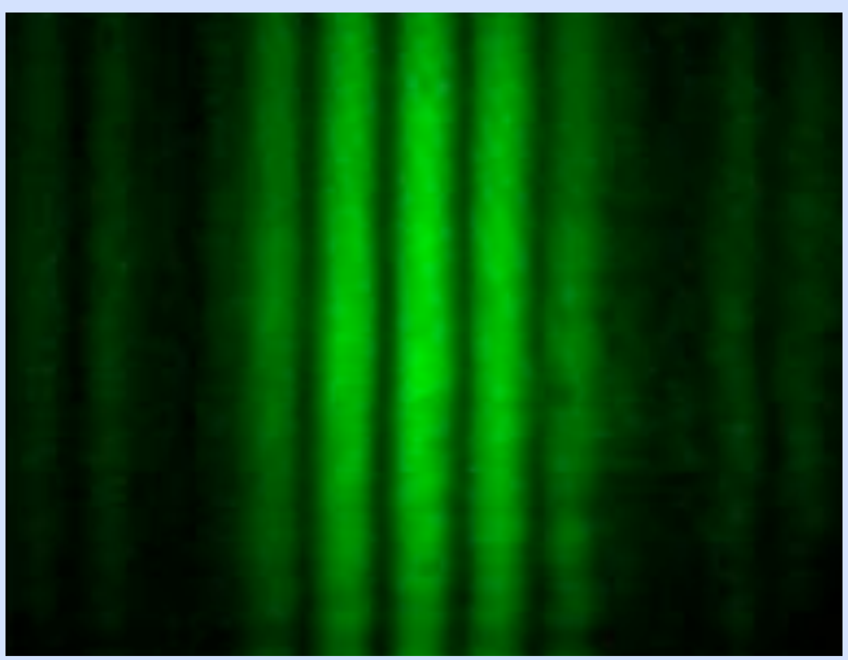
Metrology

Control of internal states

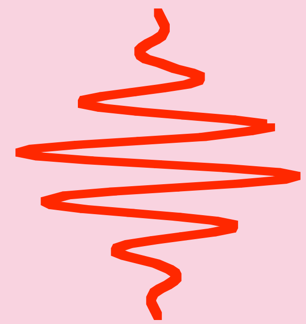
Quantum mechanics: interference

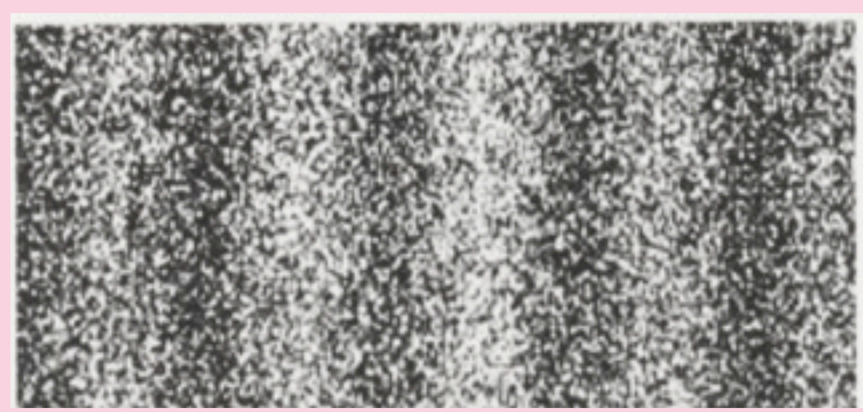
Light
massless


=


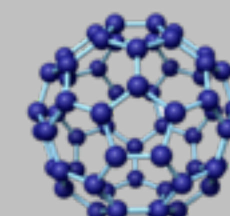


Electrons
m = 9.8 x 10^{-31} kg

e^{-} =




C-60
m = 1.2 x 10^{-24} kg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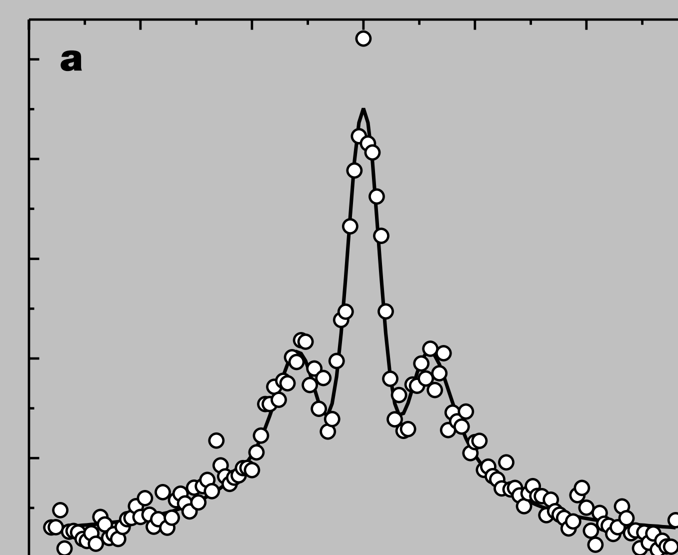
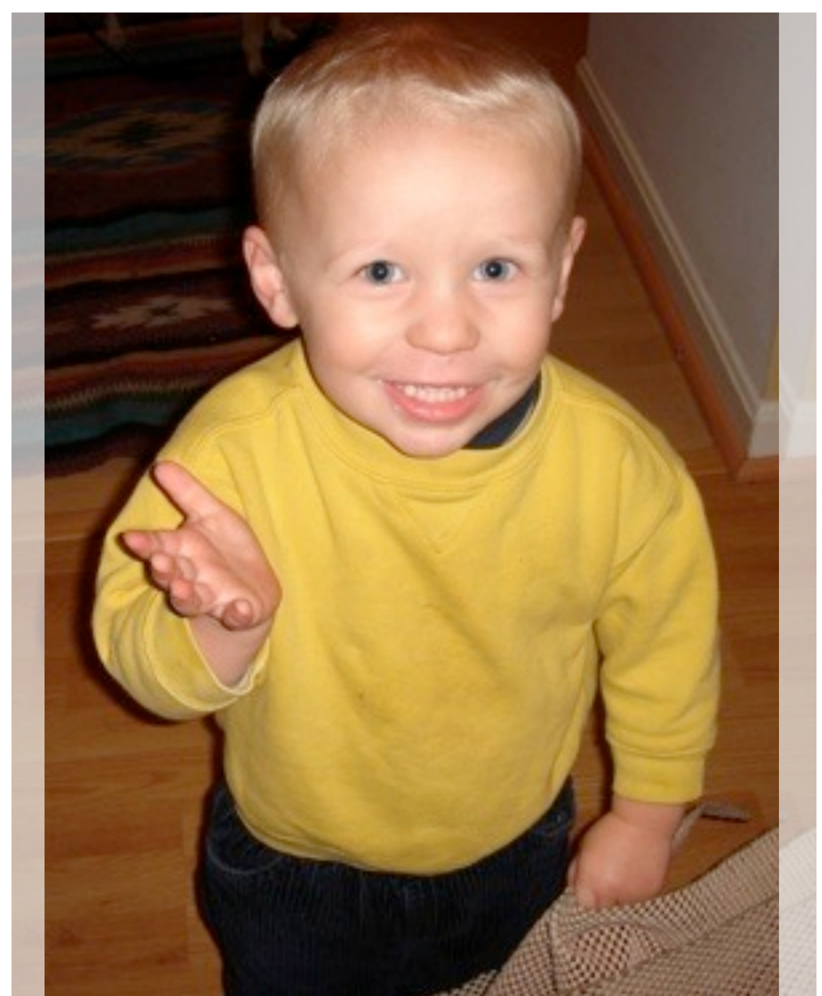


Quantum mechanics: **when** is it?

$$\Delta v = \frac{2\pi\hbar}{m\Delta x}$$

The uncertainty in the position of a 100 kg person at 1 m/s is just
7 nano-nano-nano-nano meters (*7* x 10^{-36} meters).

For people this just doesn't matter (even a rapidly moving 15 kg one)

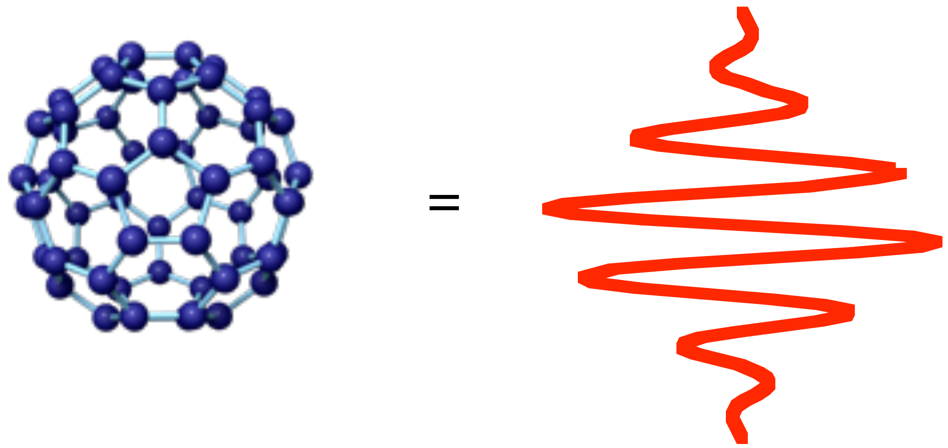


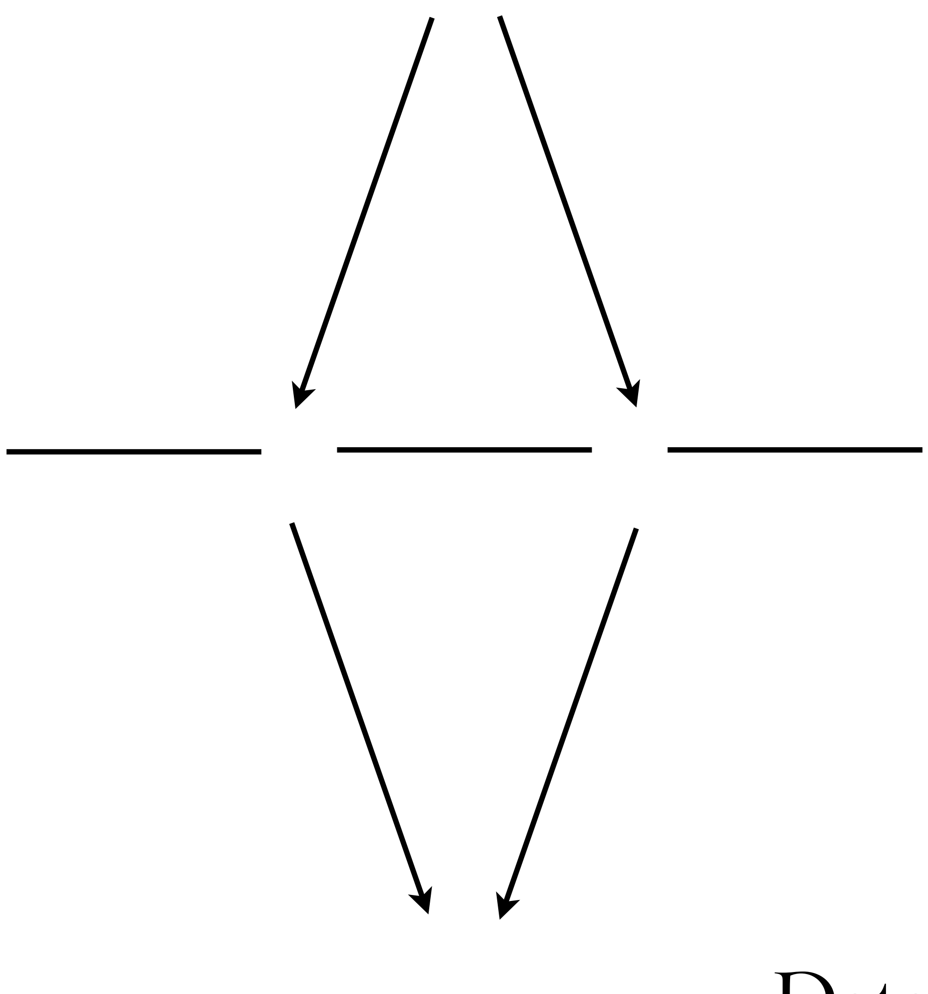
Back to quantum mechanics

Randomness

Quantum mechanics is a full deterministic theory (no randomness) **until** measurement

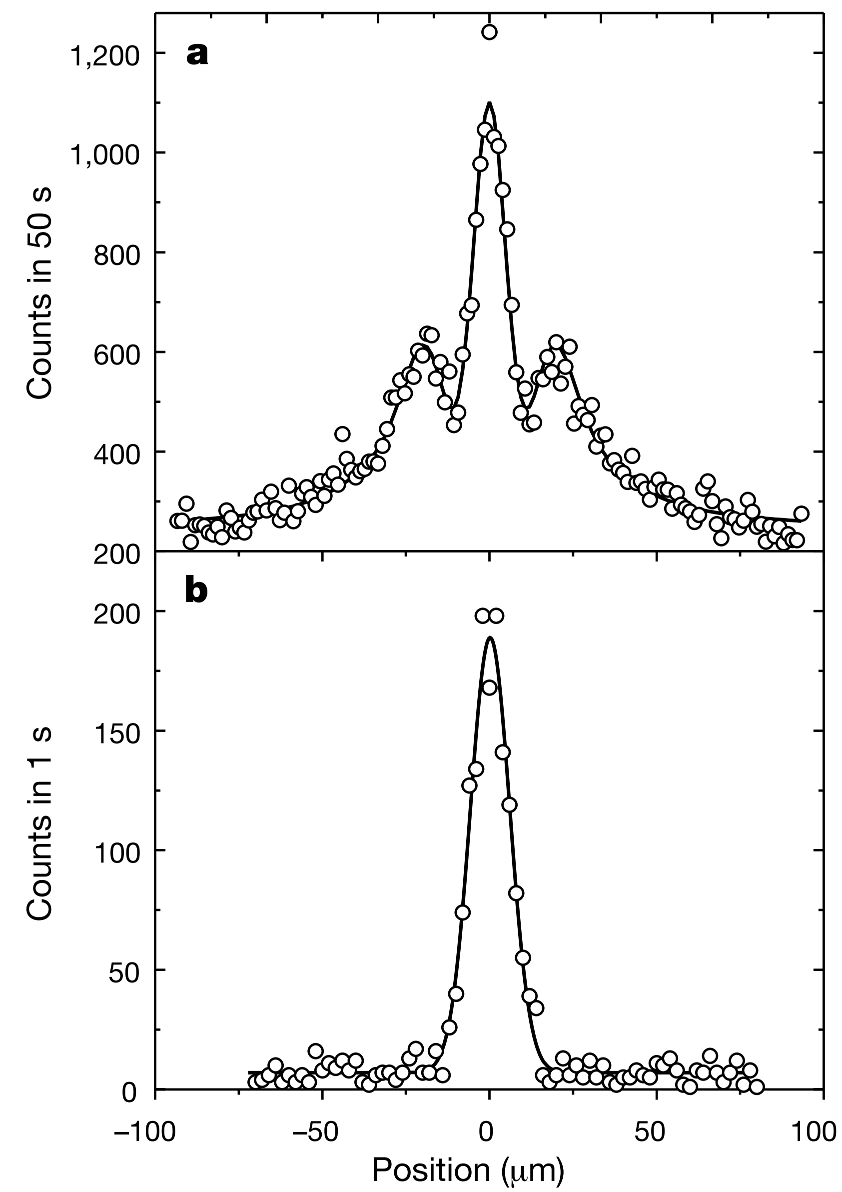
Quantum mechanics: interference (IV)





Detector

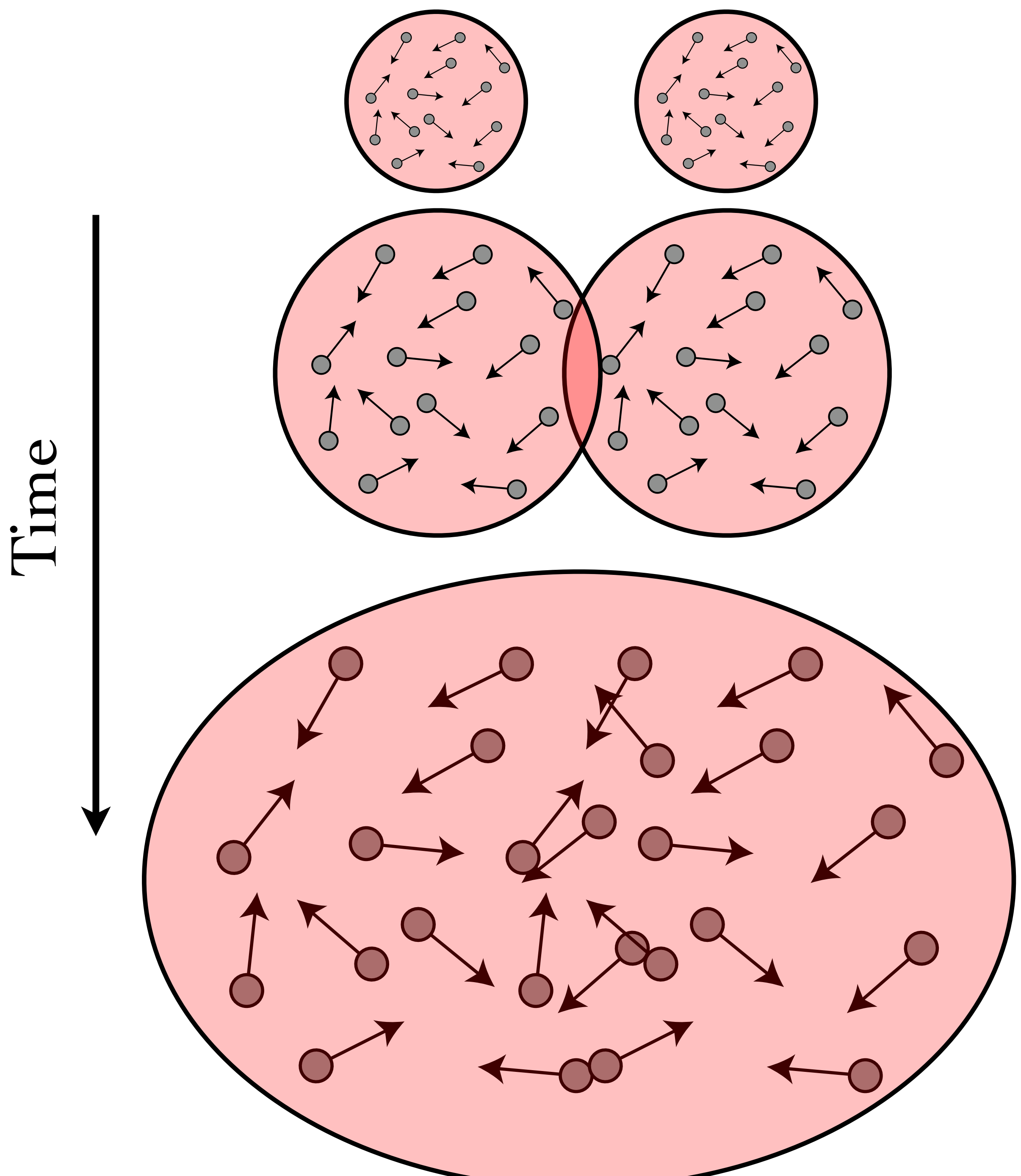
Interference of *individual* buckyballs.
One at a time.



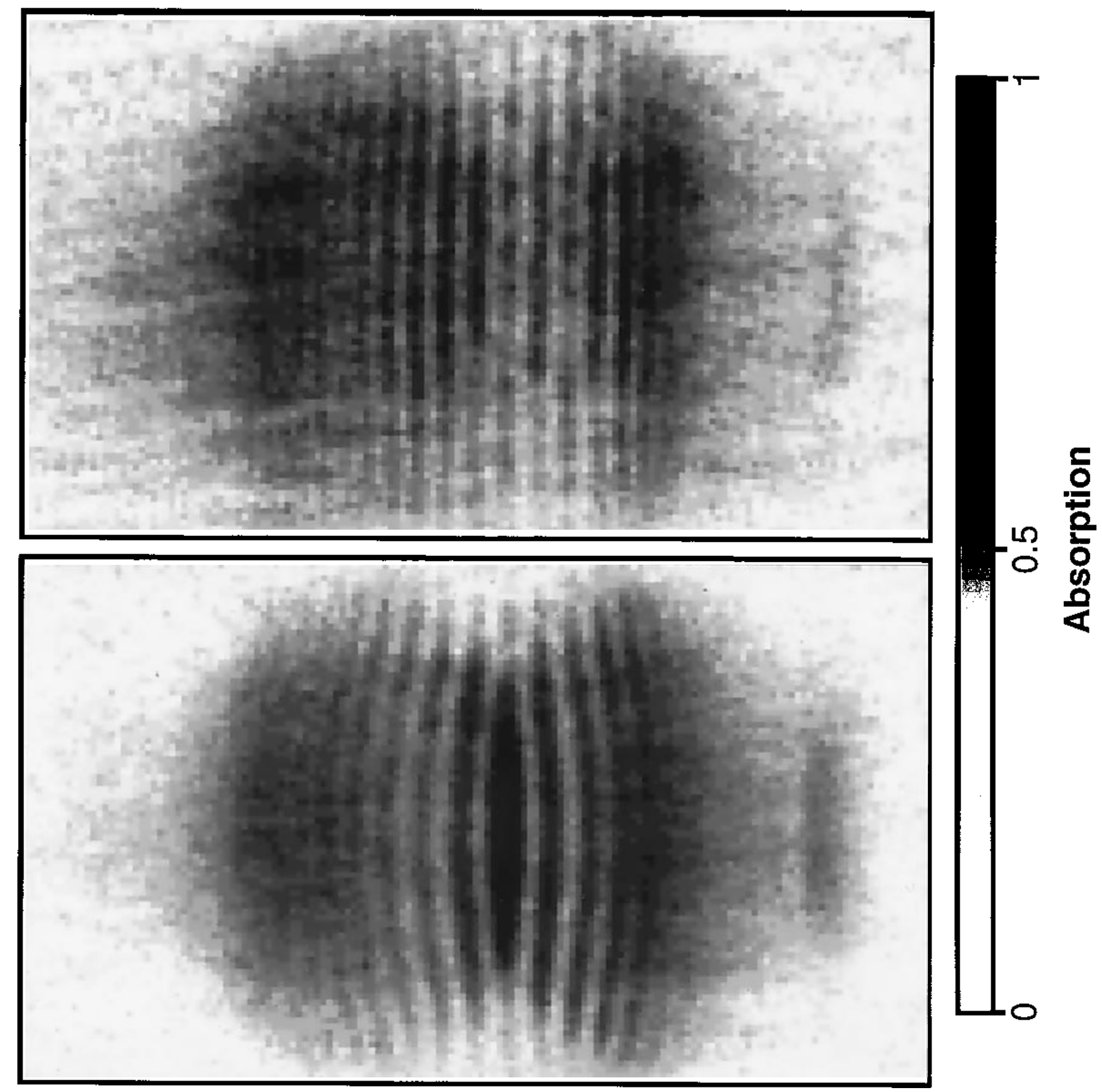
Arndt et al. Nature (1999)

Quantum mechanics: interference (II)

Two usual gases



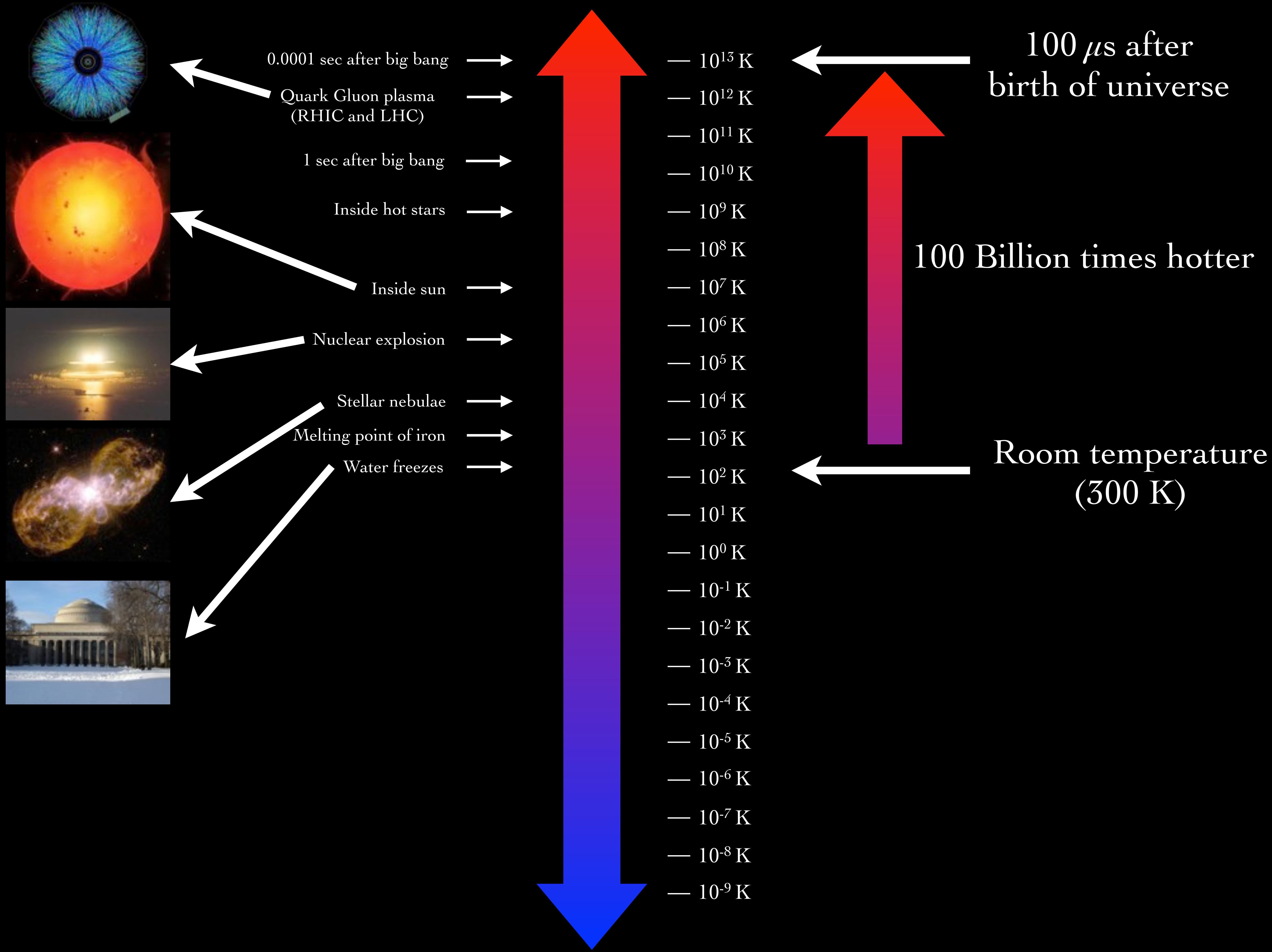
Interference between two BEC's each with 10^7 atoms



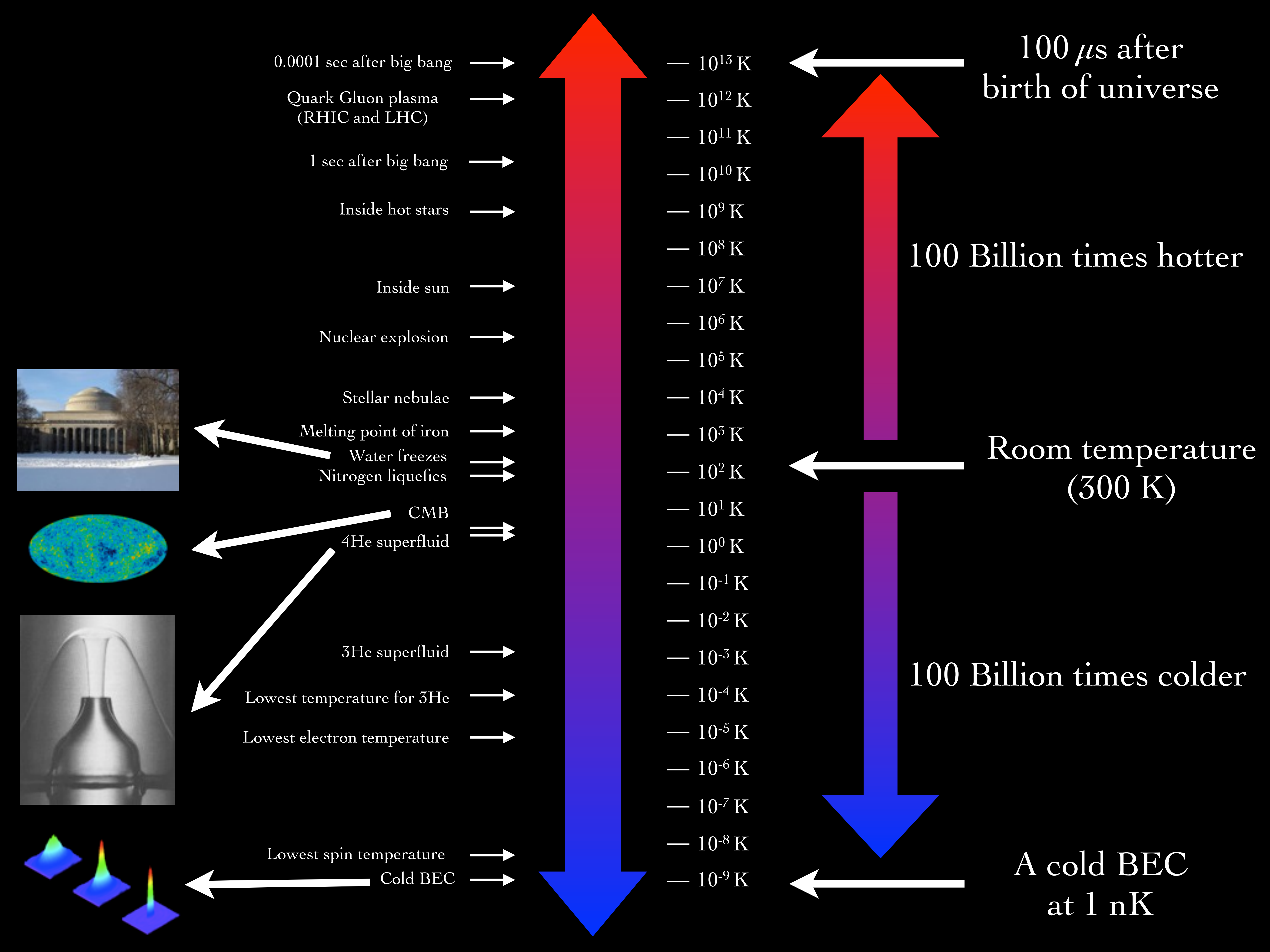
MIT: Andrews et al. Science (1997)

Mass

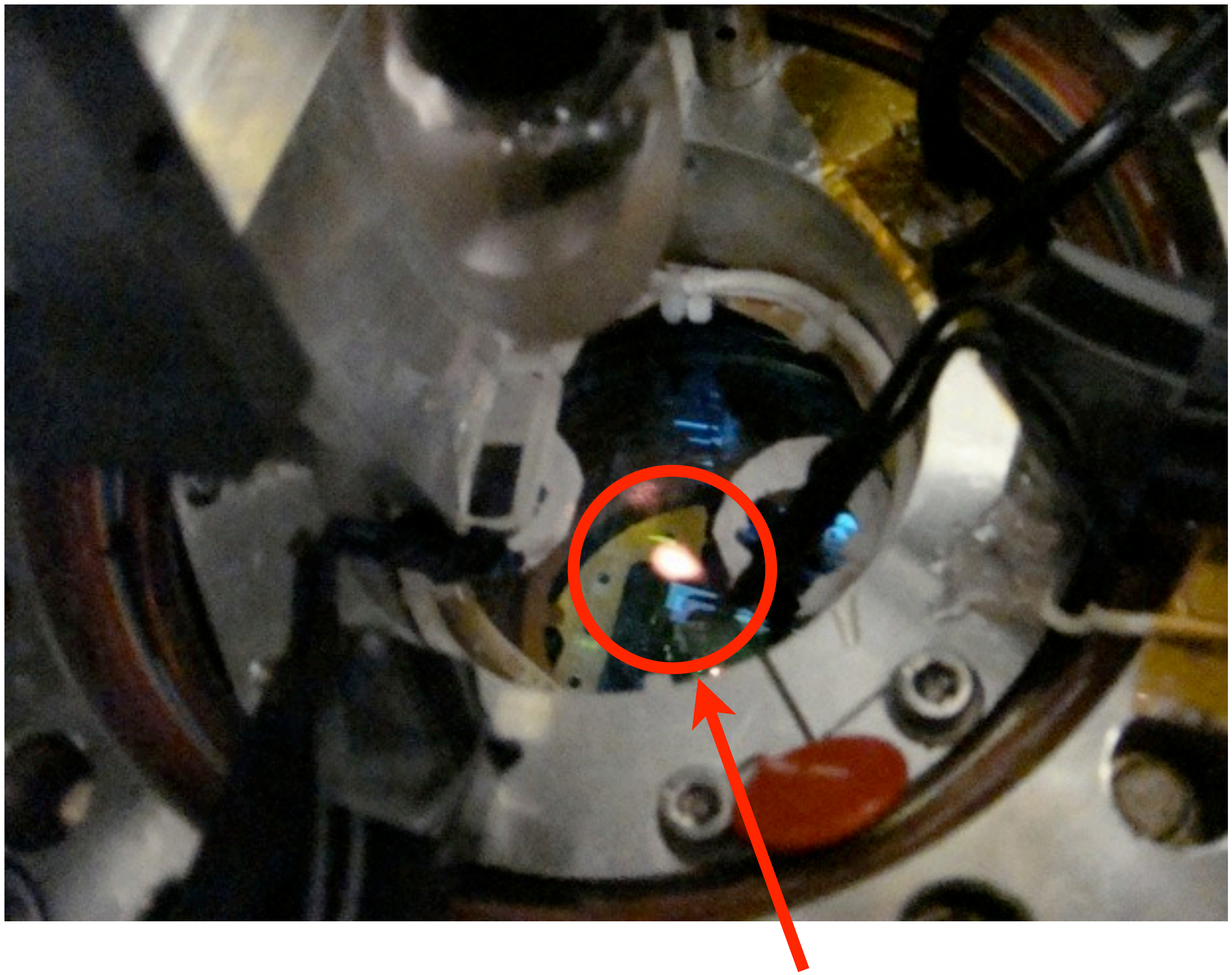
4×10^{-19} kg



0.0001 sec after big bang
Quark Gluon plasma (RHIC and LHC)
1 sec after big bang
Inside hot stars
Inside sun
Nuclear explosion
Stellar nebulae
Melting point of iron
Water freezes
— 10^{13} K
— 10^{12} K
— 10^{11} K
— 10^{10} K
— 10^{9} K
— 10^{8} K
— 10^{7} K
— 10^{6} K
— 10^{5} K
— 10^{4} K
— 10^{3} K
— 10^{2} K
— 10^{1} K
— 10^{0} K
— 10^{-1} K
— 10^{-2} K
— 10^{-3} K
— 10^{-4} K
— 10^{-5} K
— 10^{-6} K
— 10^{-7} K
— 10^{-8} K
— 10^{-9} K
100 μs after birth of universe
100 Billion times hotter
Room temperature (300 K)


0.0001 sec after big bang
Quark Gluon plasma
(RHIC and LHC)
1 sec after big bang
Inside hot stars
Inside sun
Nuclear explosion
Stellar nebulae
Melting point of iron
Water freezes
Nitrogen liquefies
CMB
4He superfluid
3He superfluid
Lowest temperature for 3He
Lowest electron temperature
Lowest spin temperature
Cold BEC
— 10^{13} K
— 10^{12} K
— 10^{11} K
— 10^{10} K
— 10^{9} K
— 10^{8} K
— 10^{7} K
— 10^{6} K
— 10^{5} K
— 10^{4} K
— 10^{3} K
— 10^{2} K
— 10^{1} K
— 10^{0} K
— 10^{-1} K
— 10^{-2} K
— 10^{-3} K
— 10^{-4} K
— 10^{-5} K
— 10^{-6} K
— 10^{-7} K
— 10^{-8} K
— 10^{-9} K
100 μs after
birth of universe
100 Billion times hotter
Room temperature
(300 K)
100 Billion times colder
A cold BEC
at 1 nK

Anatomy of an experiment: laser cooling

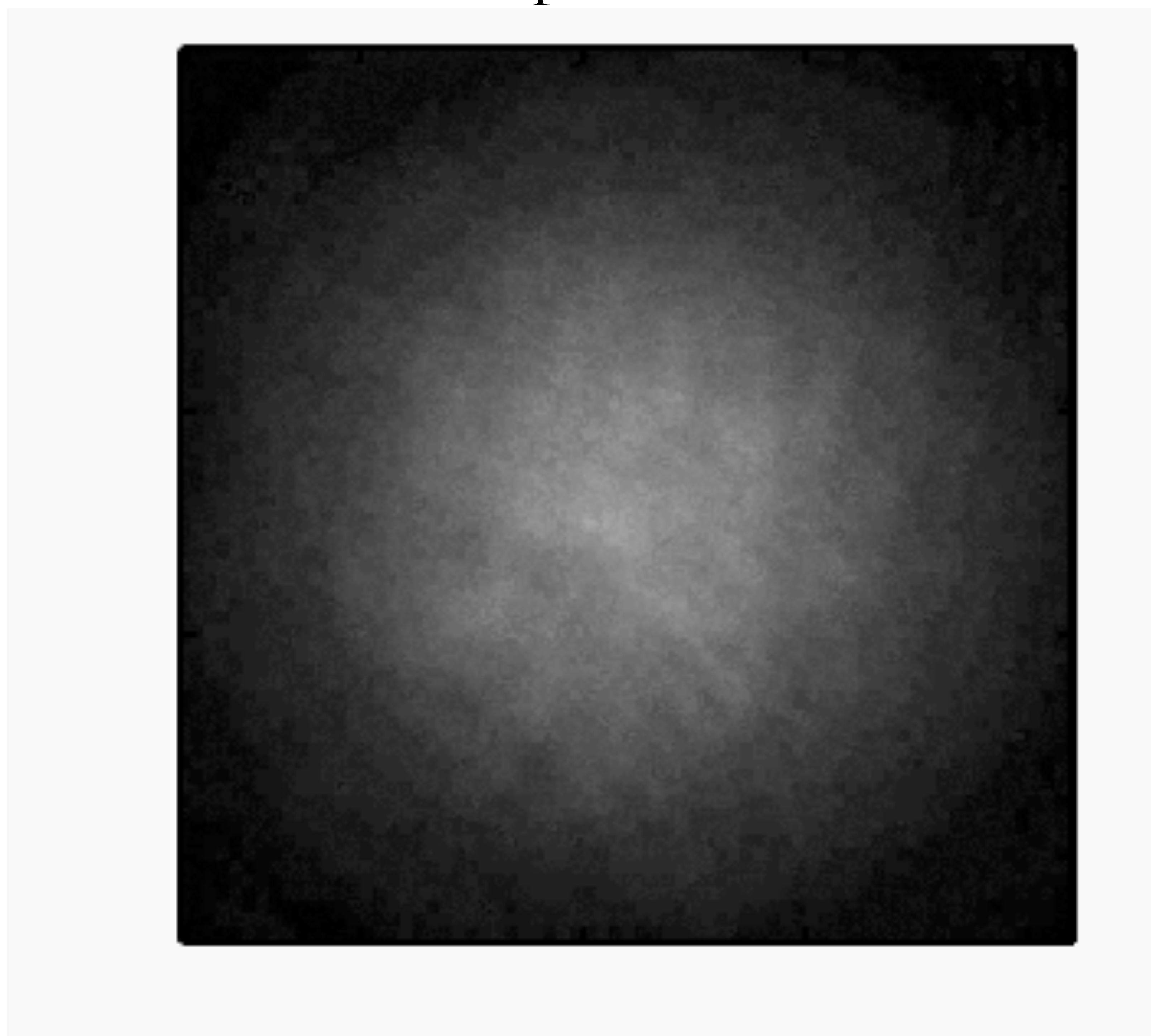


~3 billion laser cooled atoms
$T \sim 500\ \mu K$

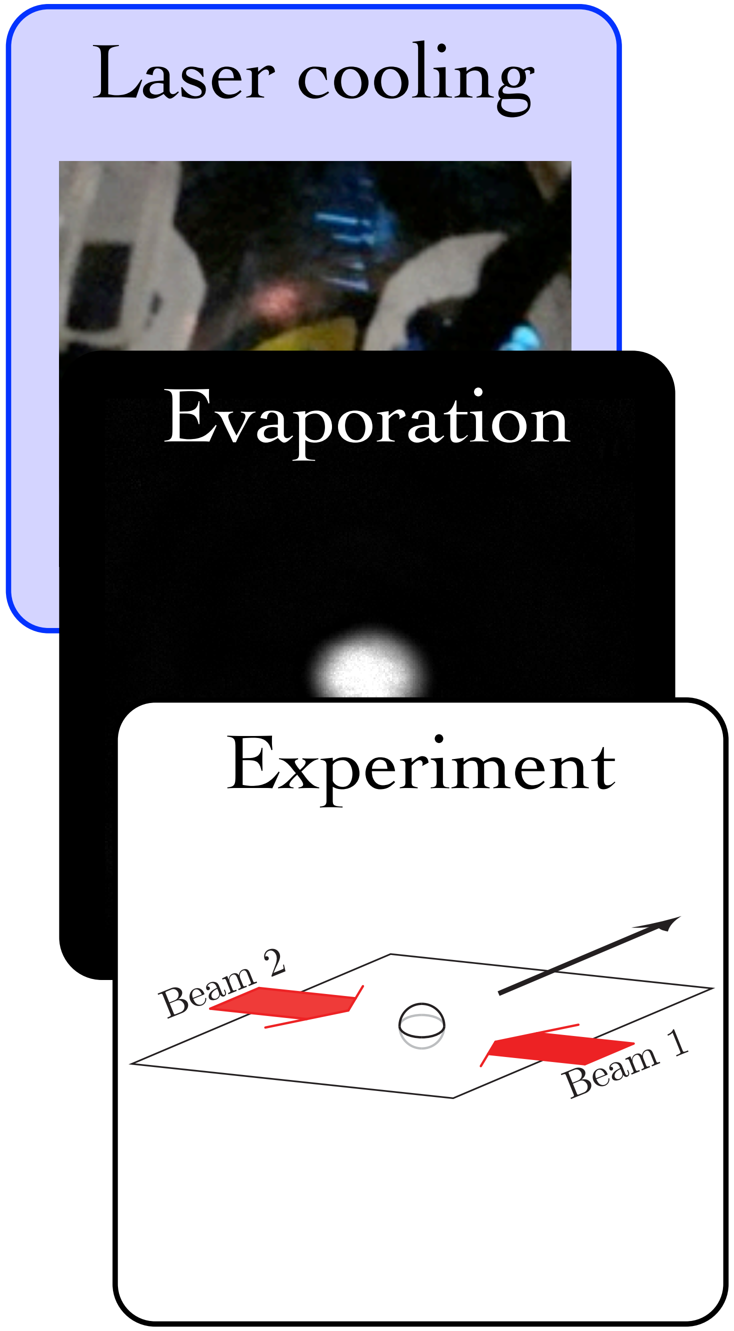
Anatomy of an experiment: evaporation

Laser cooling

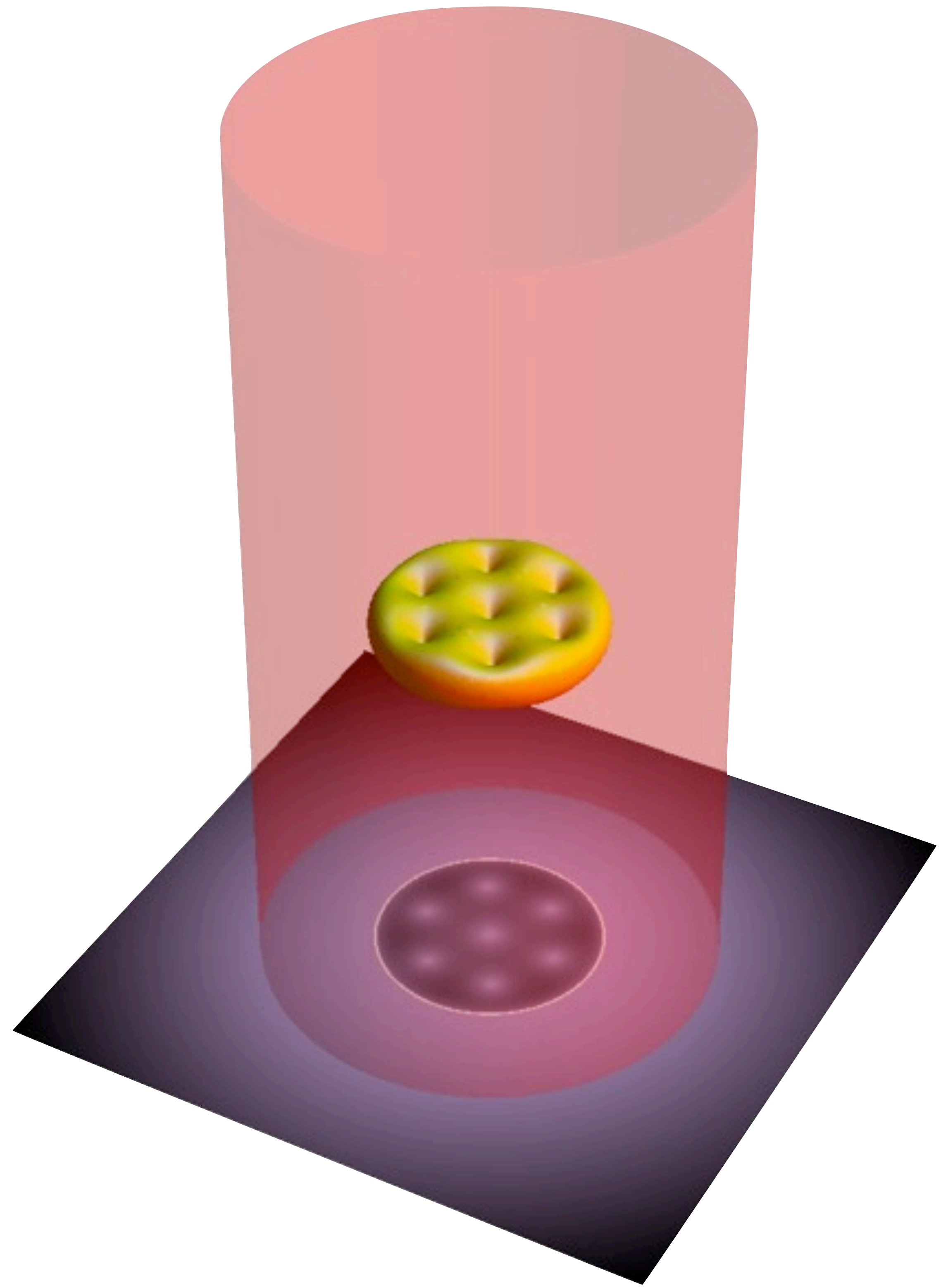
Evaporation



Anatomy of an experiment: detection

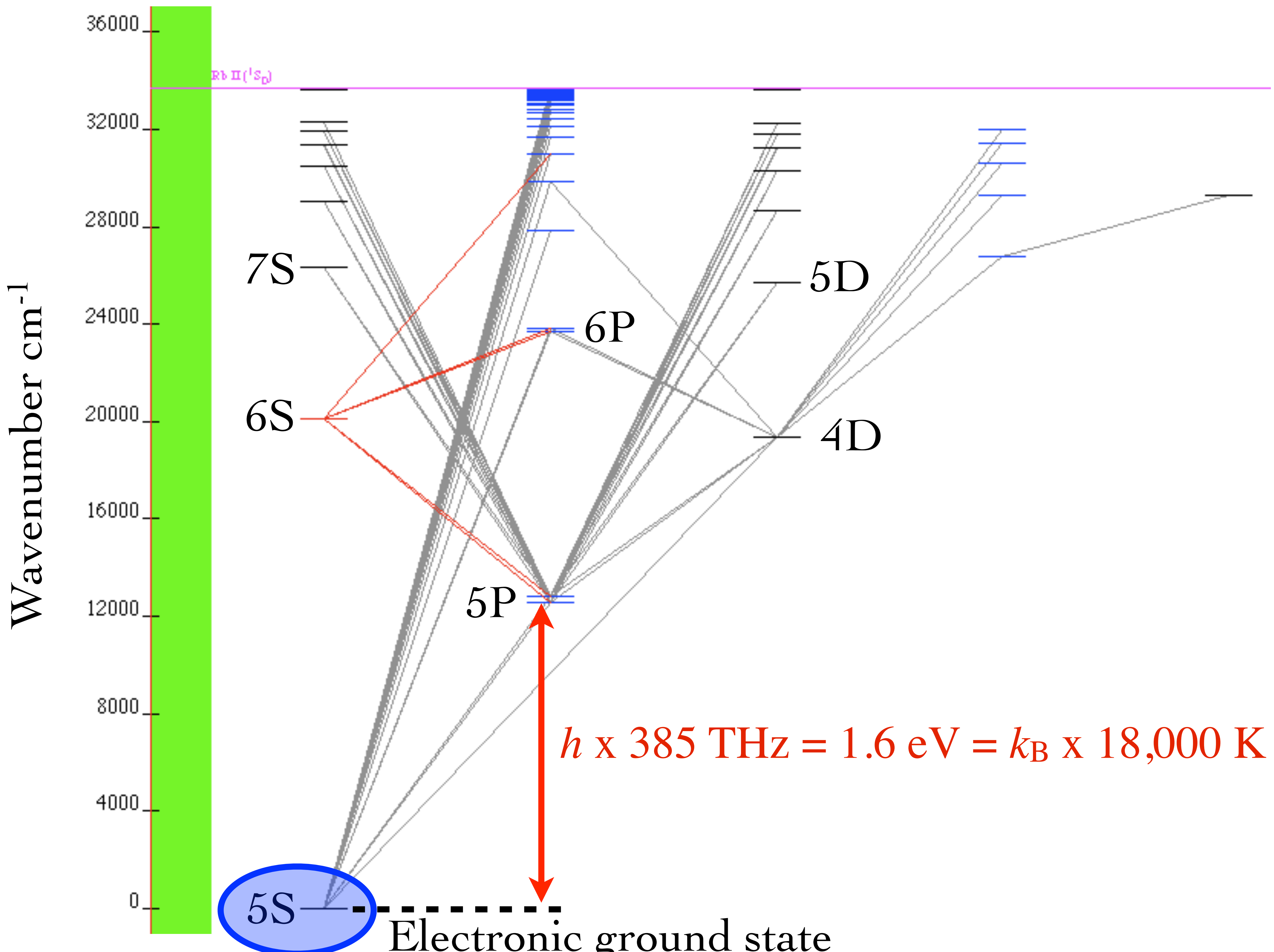


Absorption imaging

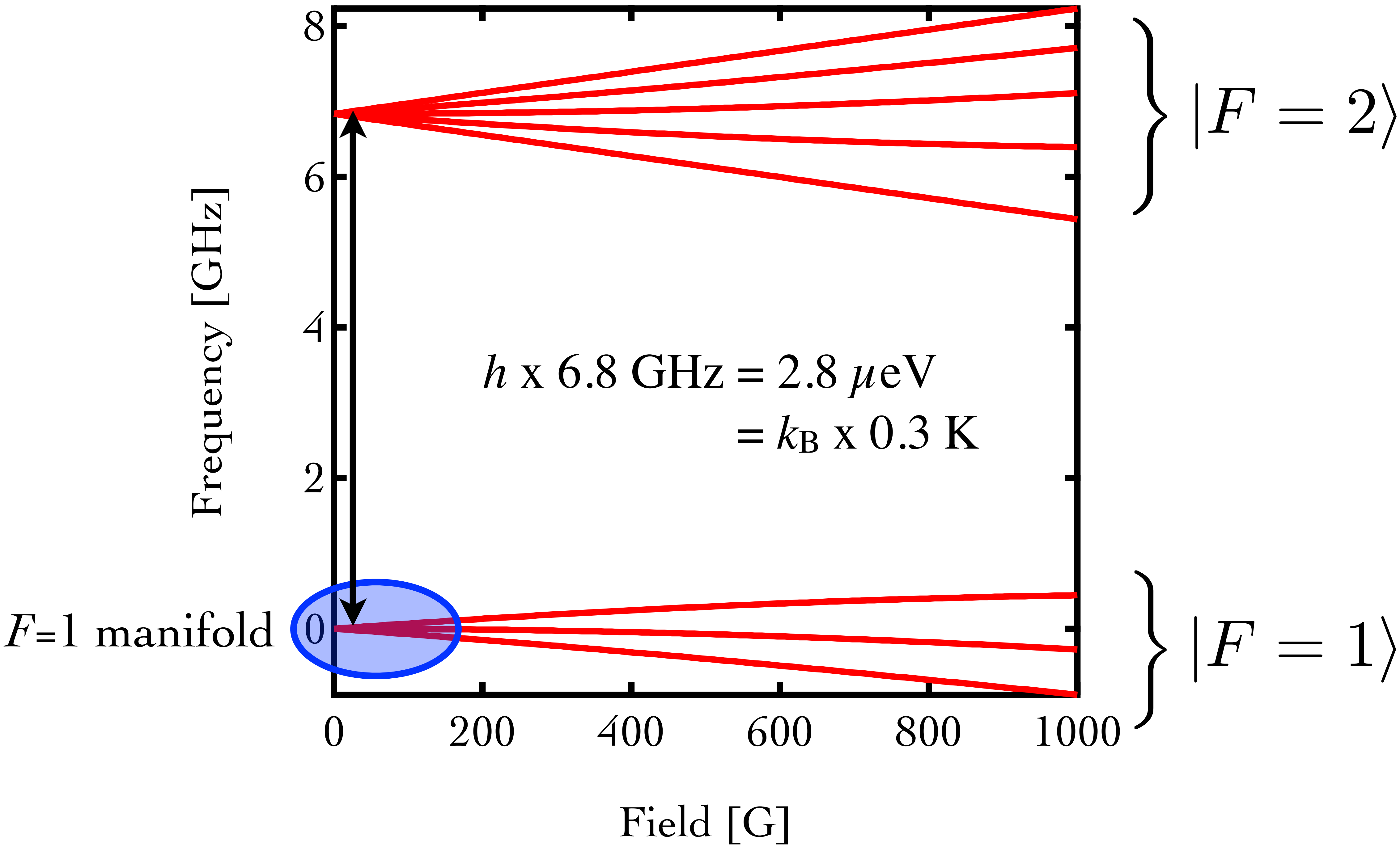


Rubidium 87: "The GaAs of atoms"

An atom is perhaps *the* quintessential quantum system



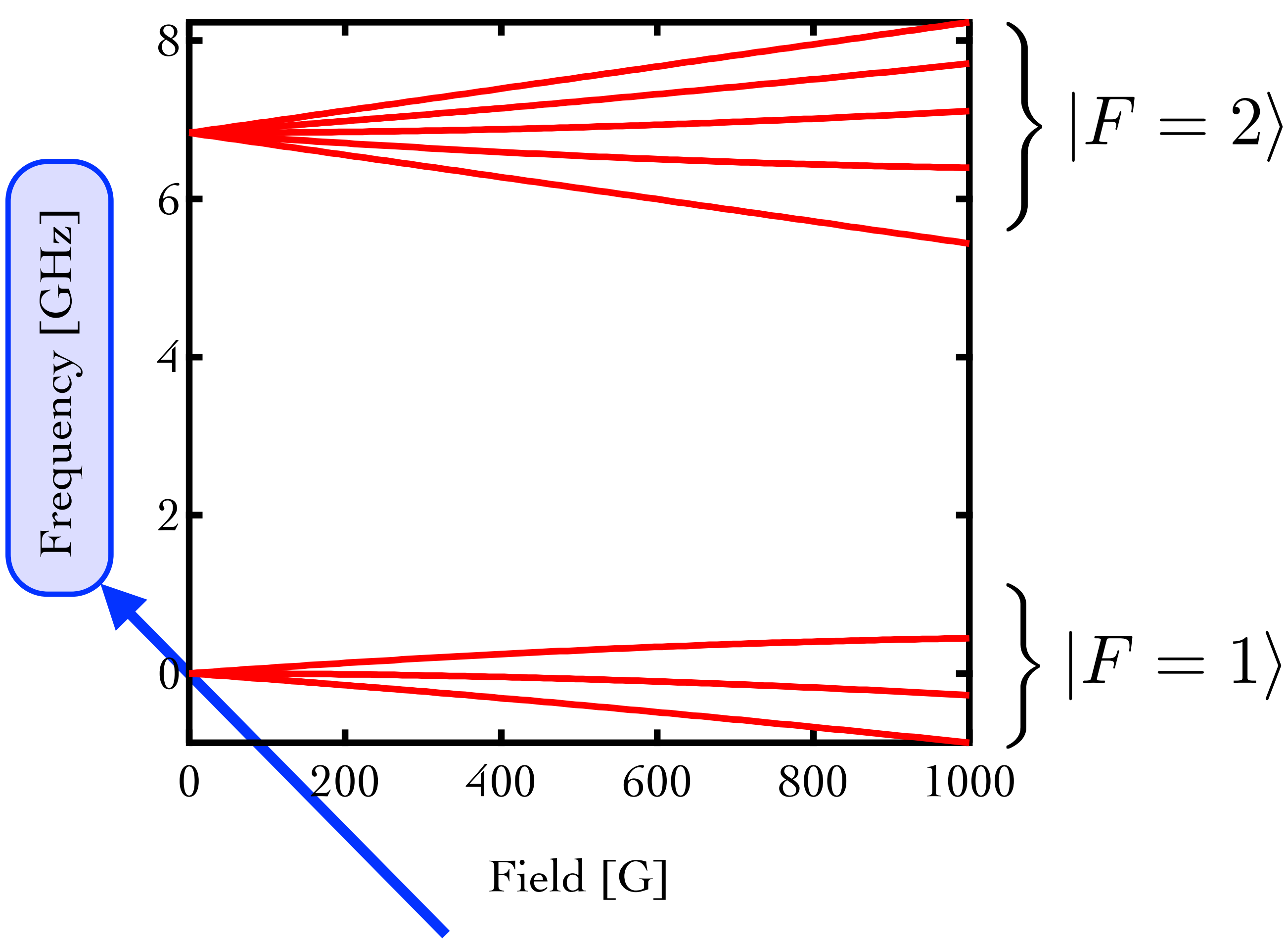
Rubidium 87: $5S_{1/2}$ ground state



What do magnetic fields do?

<u>To spins</u>

Zeeman effect, example of ^{87}Rb



Energy in dimensions of Hz

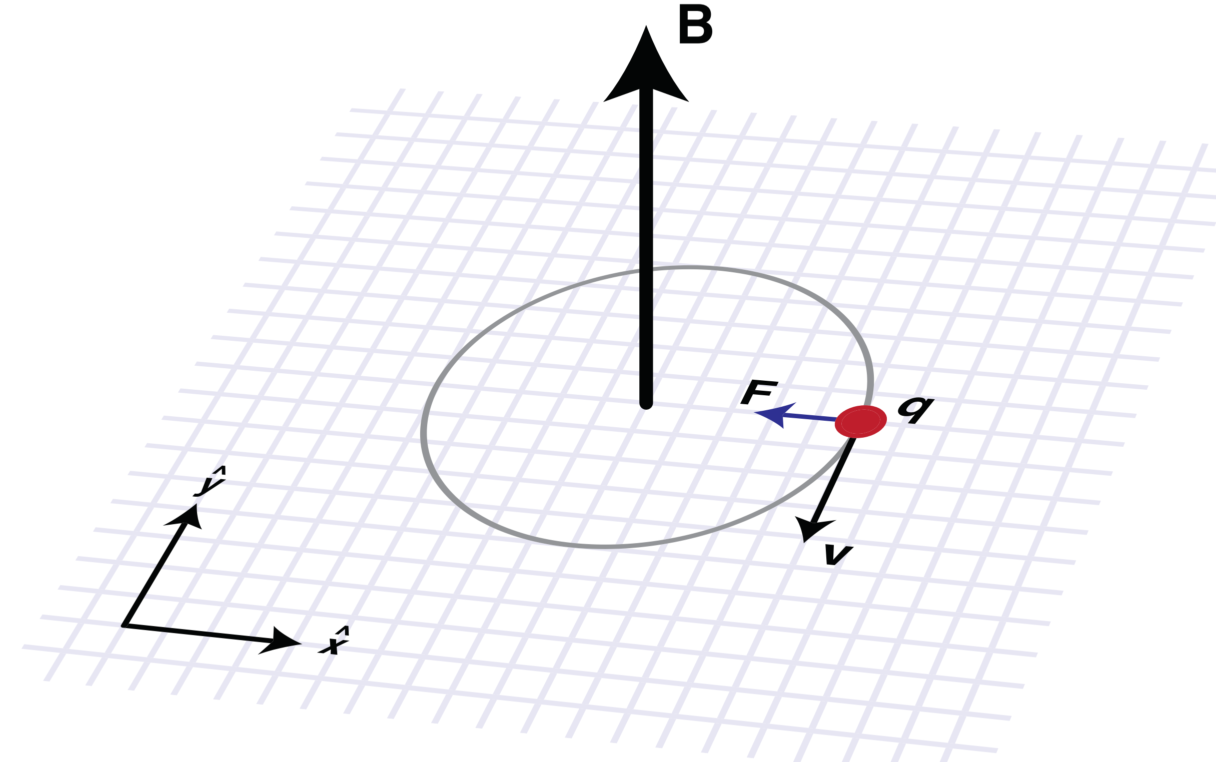
<u>To charges</u>

Lorentz force

<u>Freshman mechanics</u>

Mechanical variables and forces

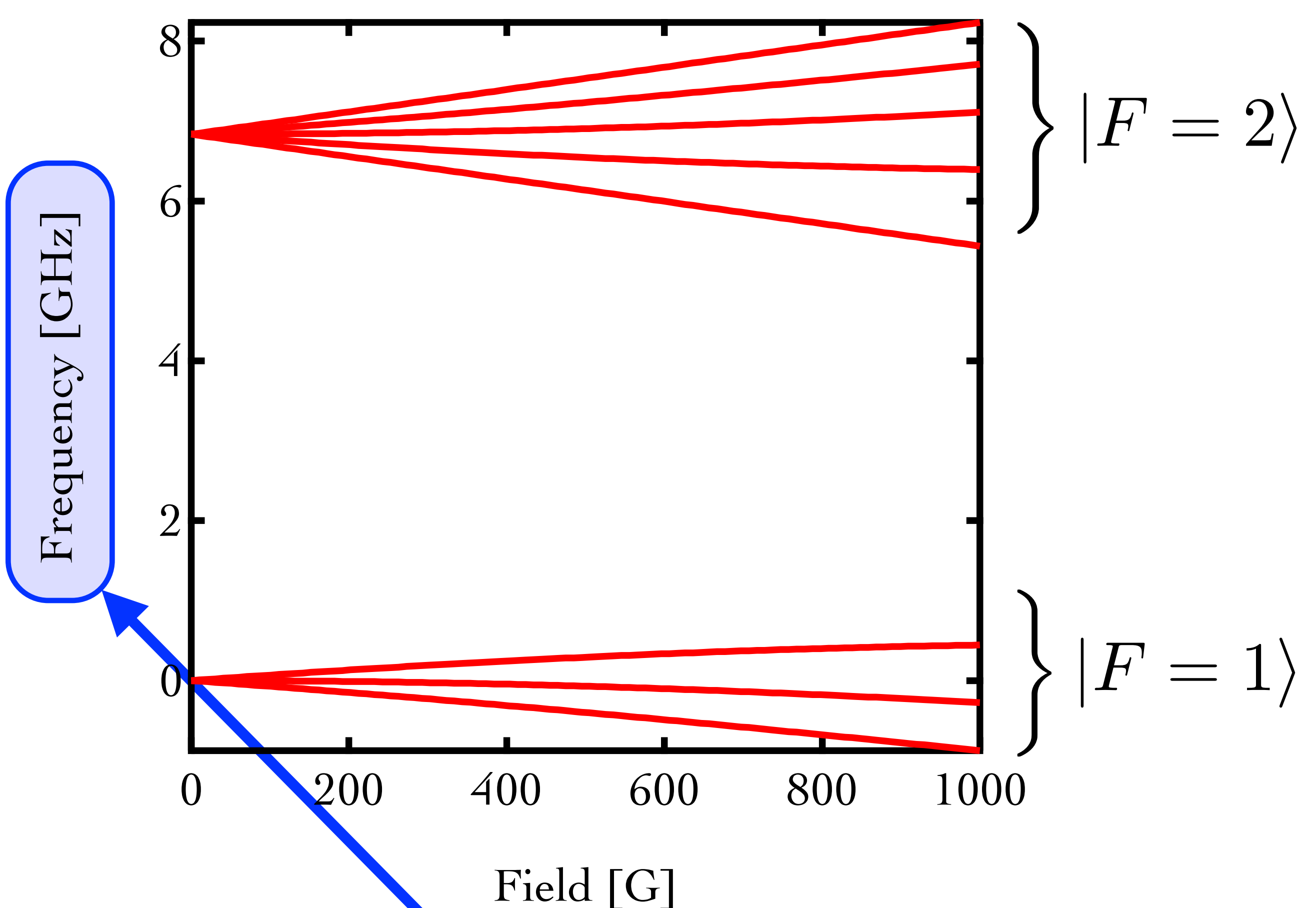
$$\mathbf{F} = q\mathbf{v} \times \mathbf{B}$$



What do magnetic fields do?

To spins

Zeeman effect, example of ^{87}Rb



Energy in dimensions of Hz

To charges

Lorentz force

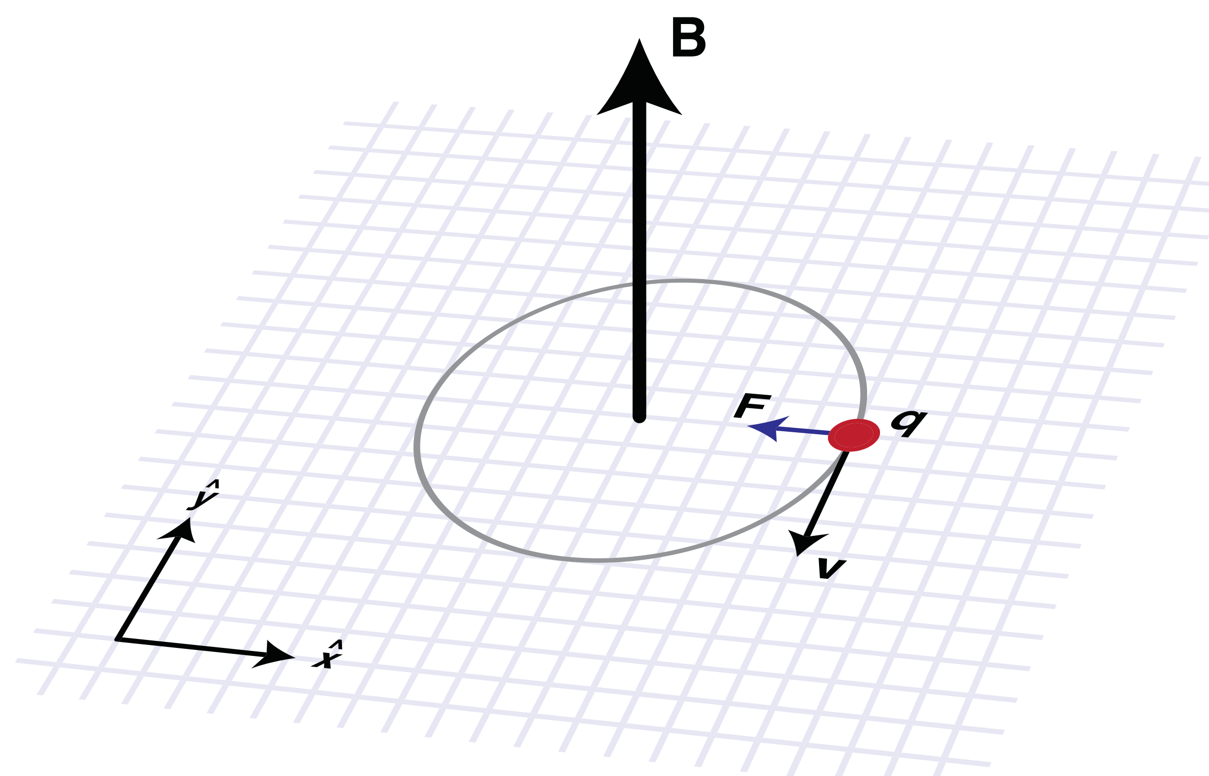
Junior mechanics

Canonical variables and vector potential

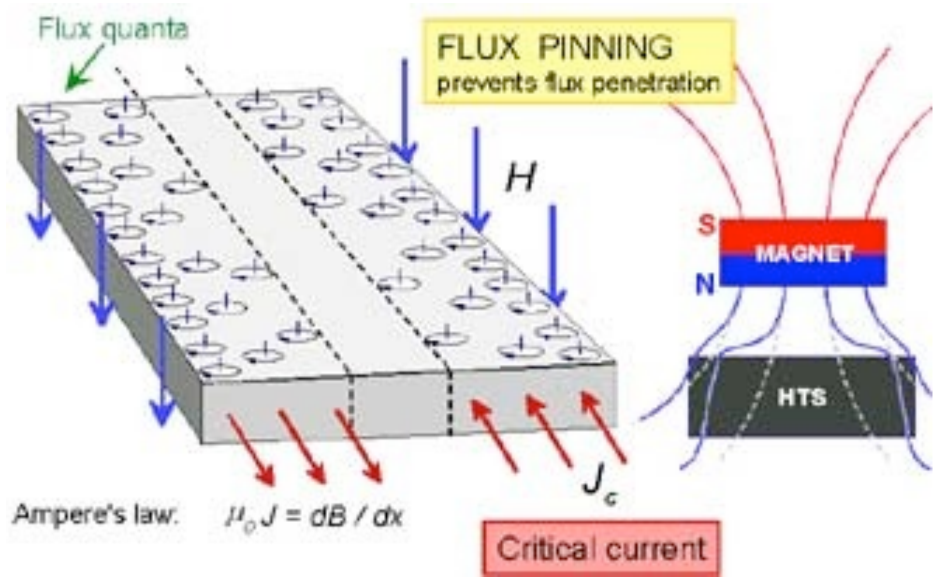
$$\mathbf{B} = \nabla \times \mathbf{A} \qquad \text{e.g.,} \quad \mathbf{A} = \frac{B}{2}(x\hat{y} - y\hat{x})$$

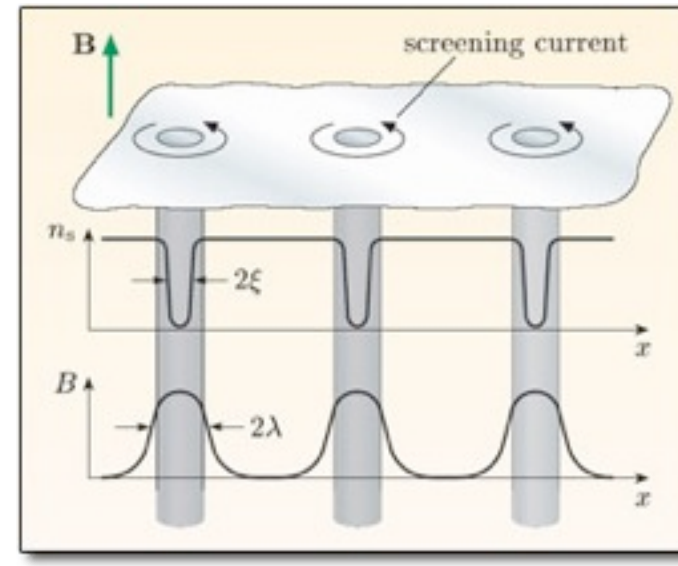
$$H = \frac{(\mathbf{p} - q\mathbf{A})^2}{2m}$$

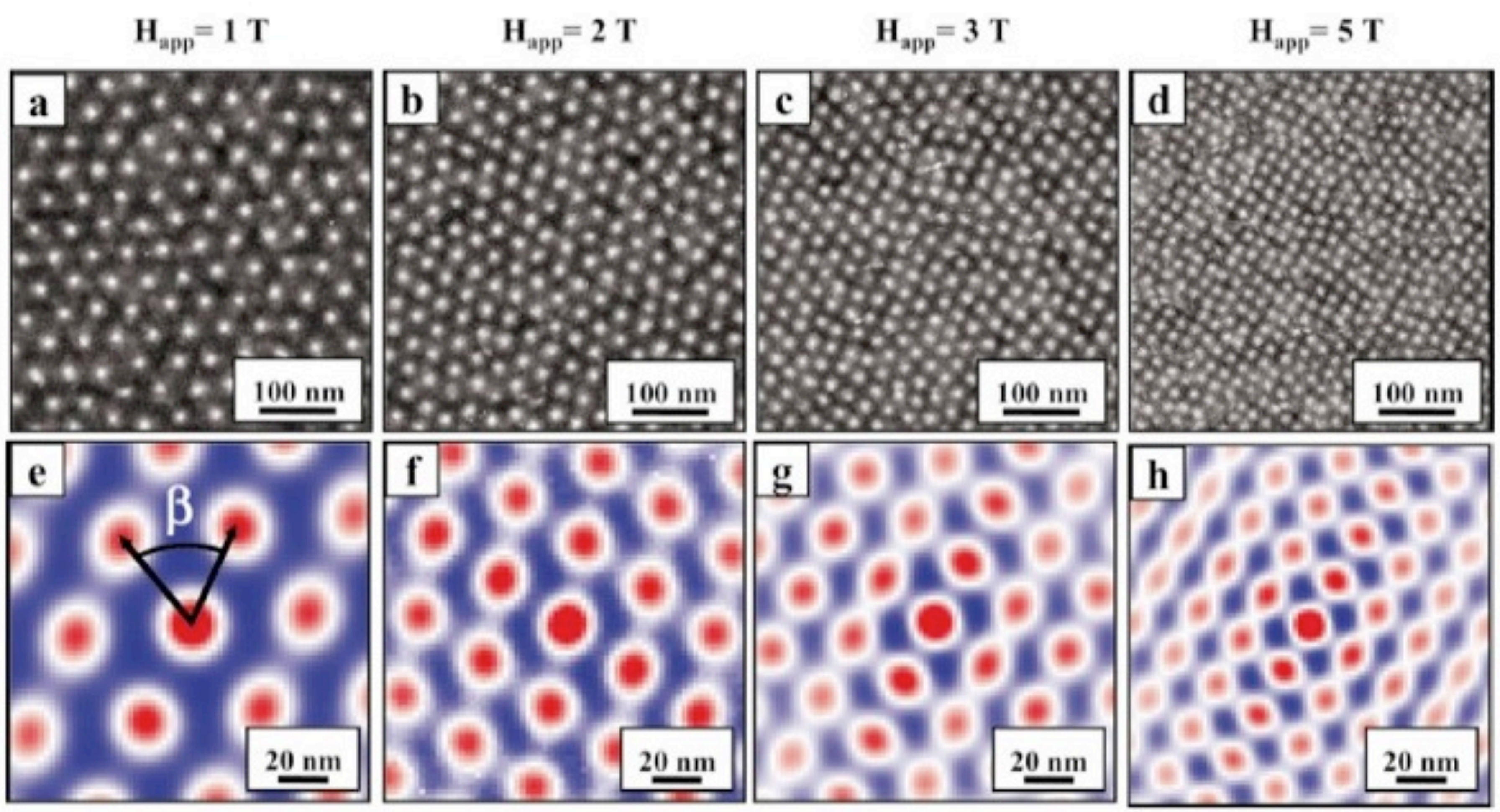
$$\dot{p}_j = -\frac{\partial}{\partial x_i}H, \ \dot{x}_j = \frac{\partial}{\partial p_i}H$$



Type-2 superconductor in a B field: vortices







References

H. Hess *et al,* PRL (1989); C. E. Sosolik *et al,* PRB (2003)

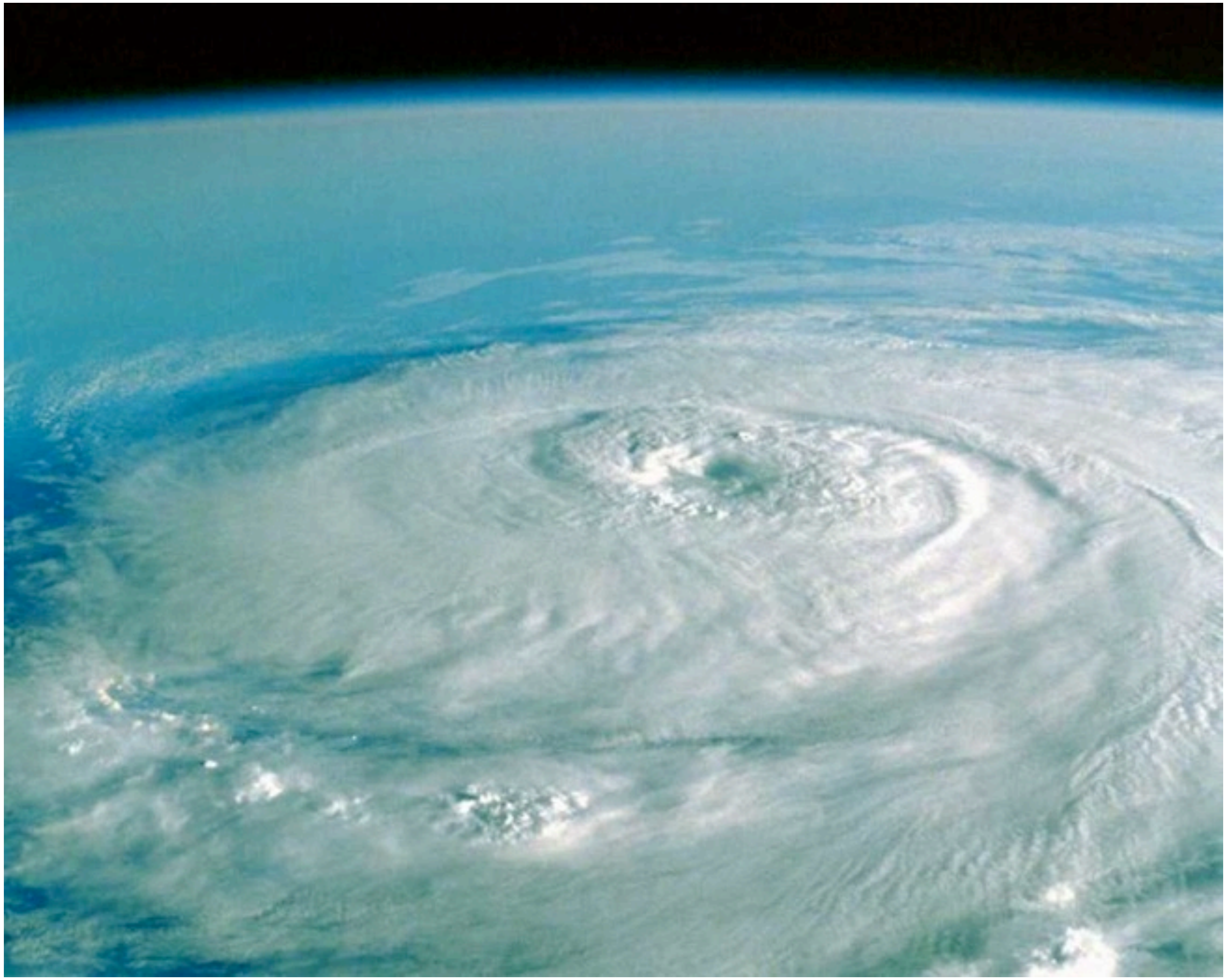
Vortices

Angular momentum is quantized in units of $\hbar$ = 1.05x10^{-34} J s.
Small!

This disposal unit has ~10^{33} quanta (10^{10} per water molecule)!

Vortices

About 10^{52} quanta (10^{14} per molecule)



Vortex: big

About 10^{95} quanta (10^{27} per hydrogen)

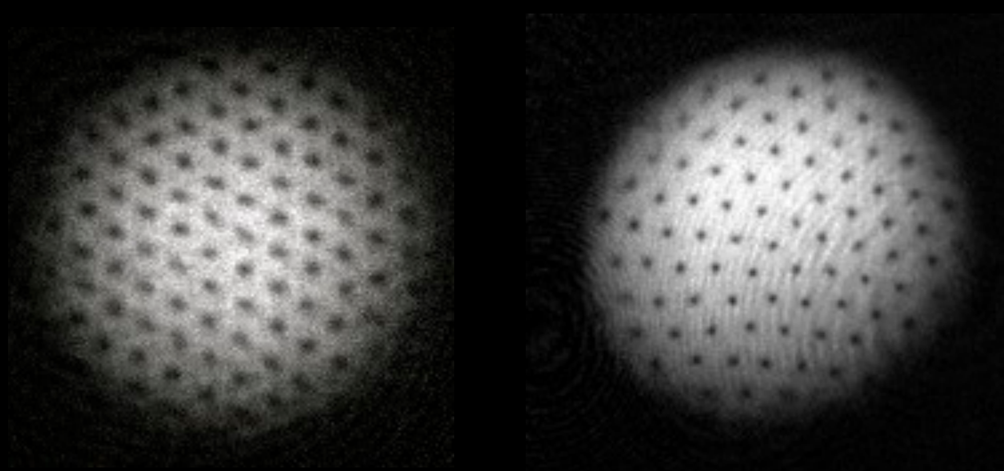
Vortex: little

10^4 quanta (1 per atom)

Simple minded example: rotation

Rotation

$$H_r = \frac{\hbar^2}{2m}\left[\left(\hat{k}'_x - \frac{m\Omega}{\hbar}y\right)^2 + \left(\hat{k}'_y + \frac{m\Omega}{\hbar}x\right)^2\right] + \frac{m\left(\omega^2 - \Omega^2\right)}{2}\mathbf{r}^2$$



JILA MIT

How to simulate magnetic fields

The Hamiltonian in the rotating frame has an effective field.

For high fields fine tuning is required.

ENS, JILA, MIT, ...

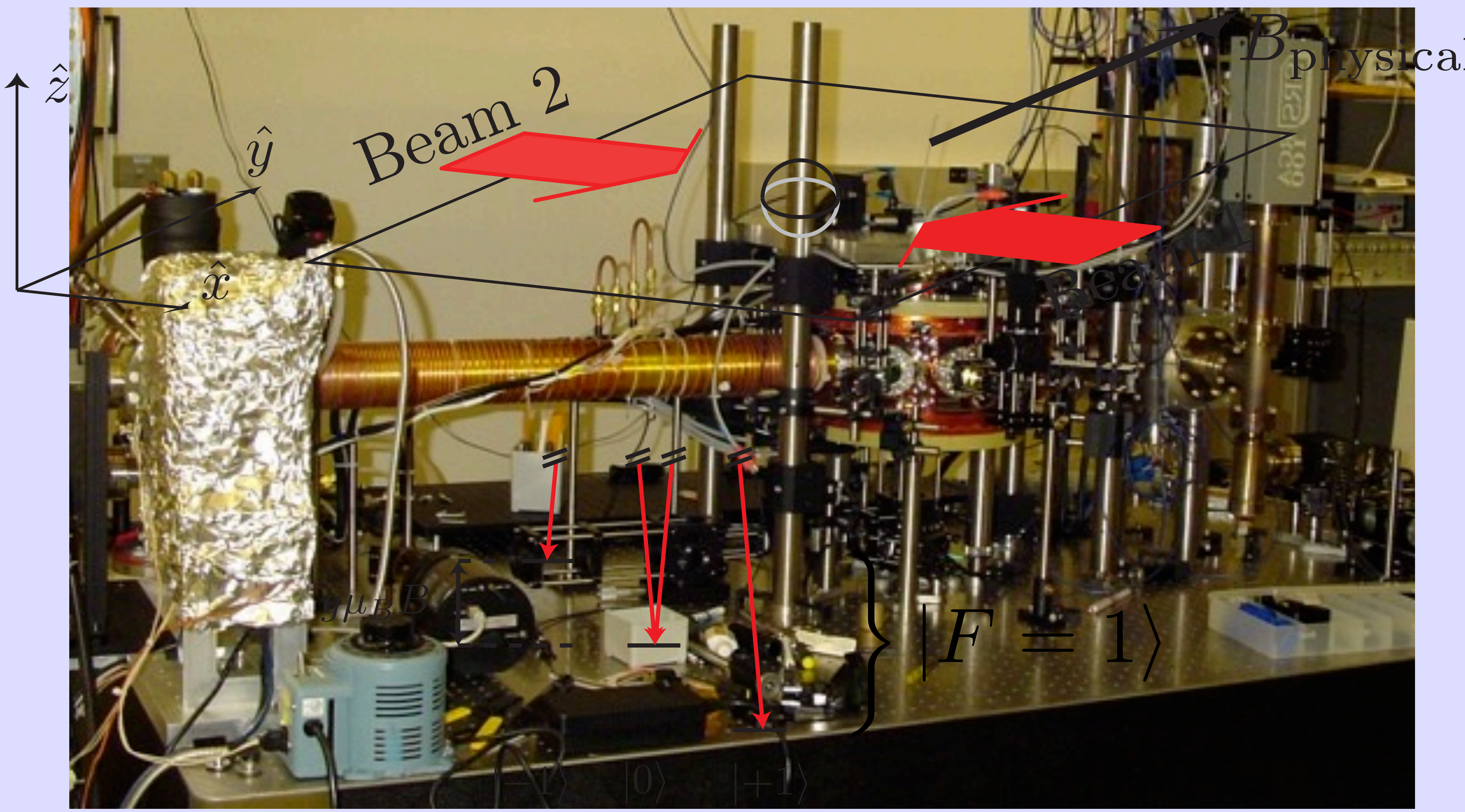
$$\hat{R}(\theta = \Omega t) = \exp\left(i\Omega t \hat{L}_z/\hbar\right)$$

$$H_r = \frac{\hbar^2}{2m}\left[\left(\hat{k}'_y + \frac{m\Omega}{\hbar}x\right)^2 + \left(\hat{k}'_x - \frac{m\Omega}{\hbar}y\right)^2\right] + \frac{m\left(\omega^2 - \Omega^2\right)}{2}\left(x^2 + y^2\right)$$

Simplicity from complexity

Raman dressed states

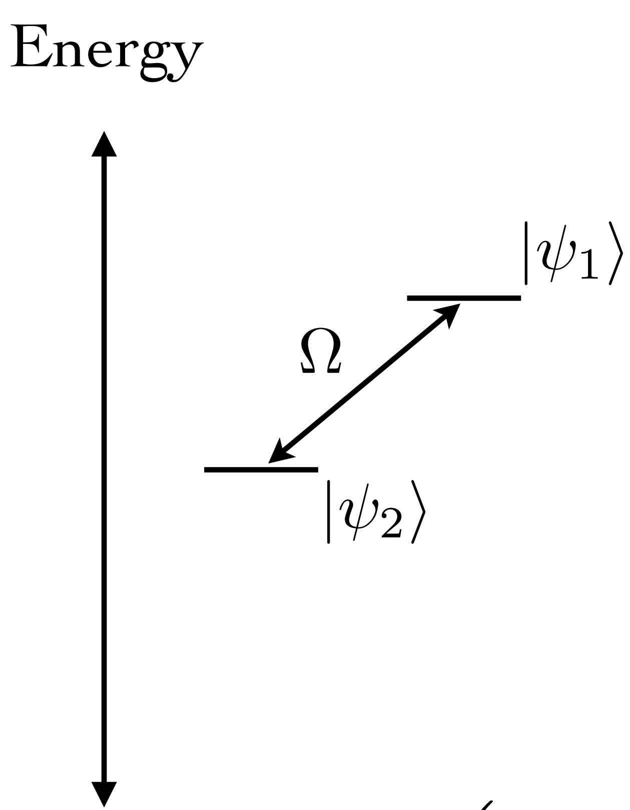
Brief description of implementation and theory



Coupled systems intuition

<u>Schematic</u>

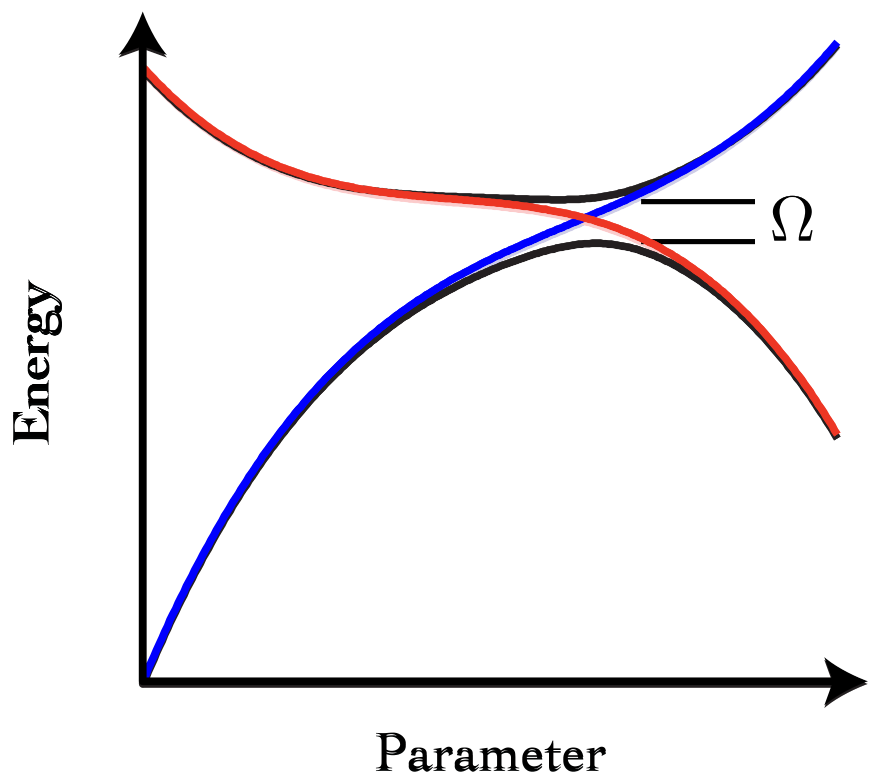
Two levels coupled together



$$H = \begin{pmatrix} E_1 & \Omega/2 \\ \Omega/2 & E_2 \end{pmatrix}$$

<u>Graphic result</u>

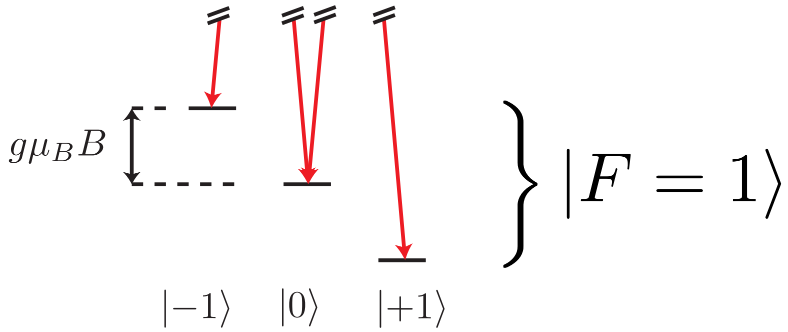
Avoided crossings

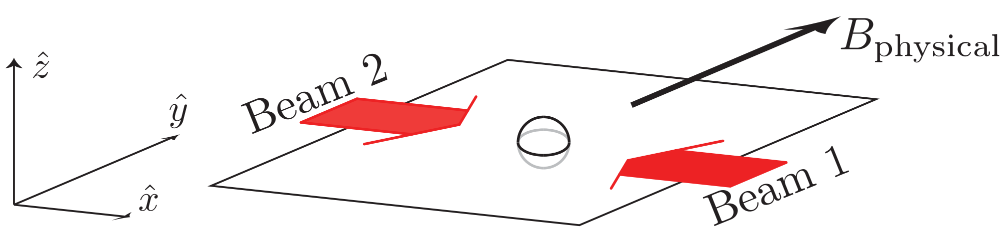


Atom light interaction: pictures

Atom light interaction

Given the following geometry and levels



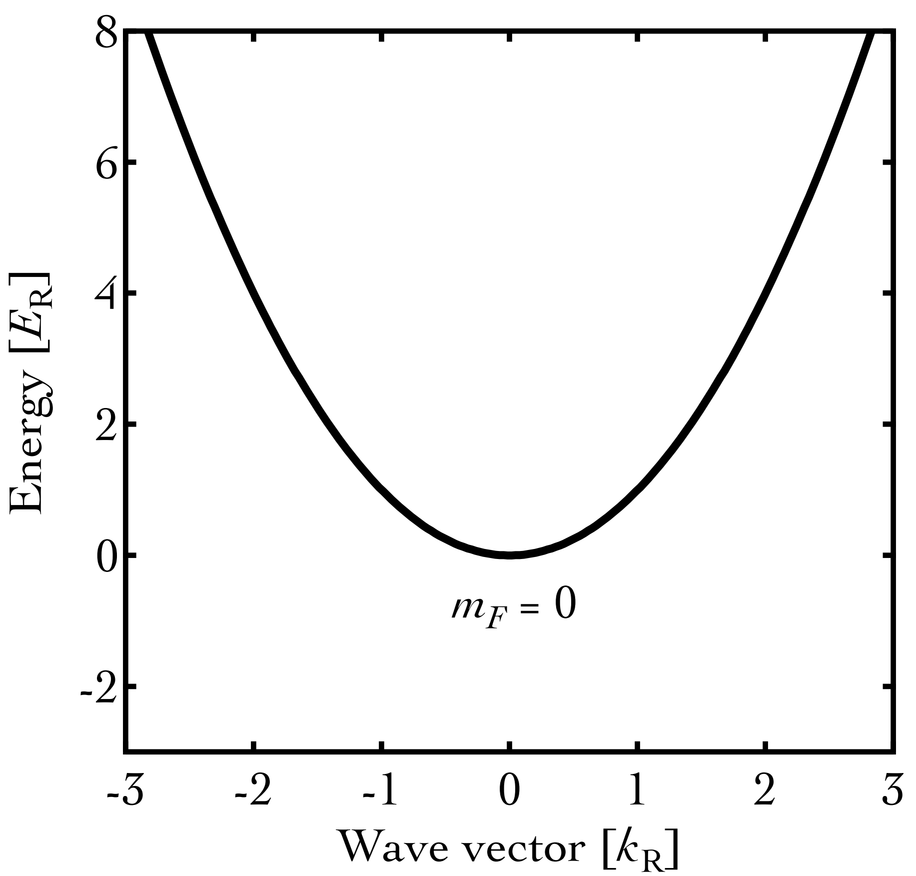


Dimensions

$$k_R = \frac{2\pi}{\lambda}, \;\; E_R = \frac{\hbar^2 k_R^2}{2m}$$

$$E_R \approx h \times 3 \text{ kHz} = k_{\mathrm{B}} \times 140 \text{ nK}$$

Coupled States



References

[1] Juzeliūnas, et al., PRA 025602 **73** (2006), + earlier pubs

[2] S.-L. Zhu, et al., PRL 240401 **97** (2006)

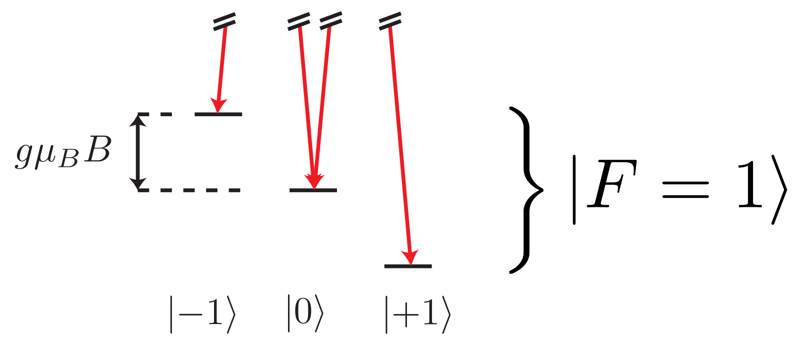
[3] Günter et al, PRA **79** 011604 (2009)

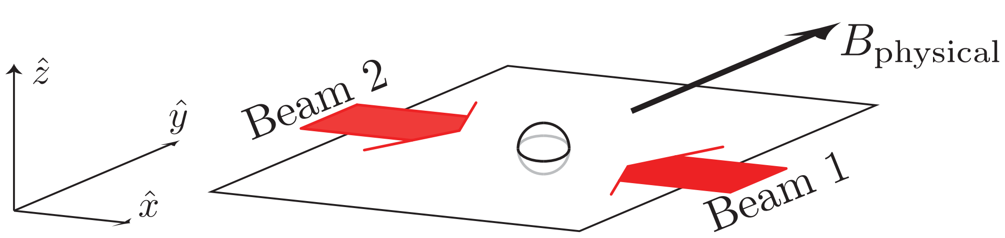
[4] IBS, PRA 063613 **79** (2009)

Atom light interaction: pictures

Atom light interaction

Given the following geometry and levels





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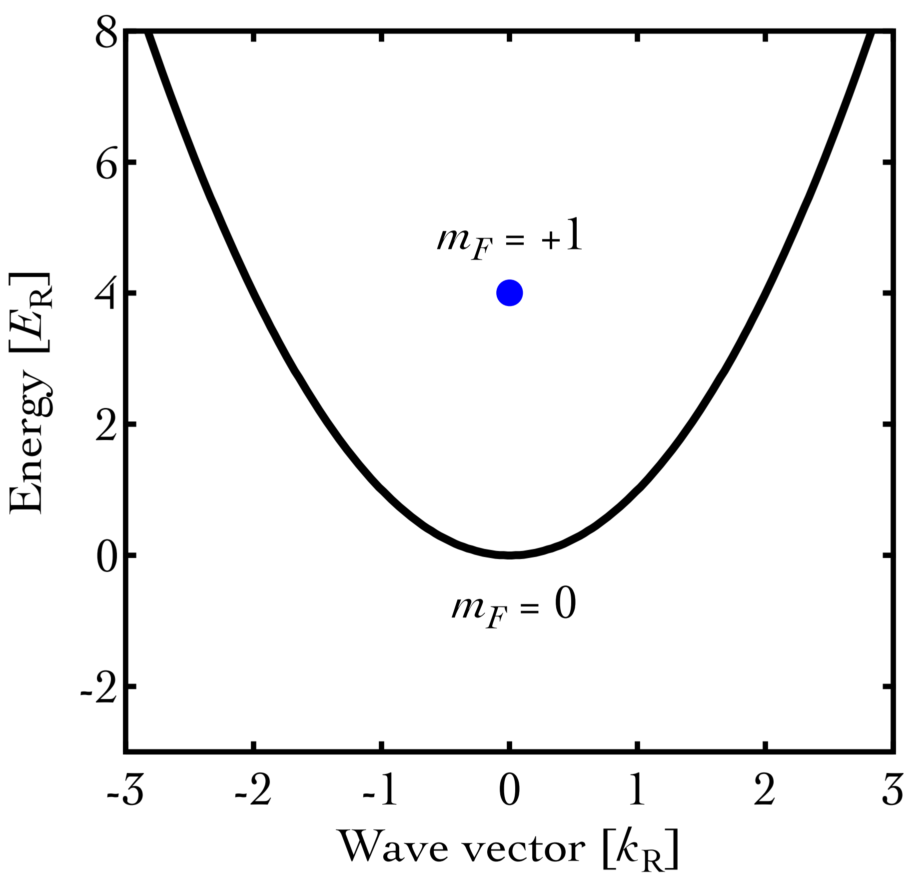
Coupled States



References

[1] Juzeliūnas, et al., PRA 025602 **73** (2006), + earlier pubs

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Atom light interaction: pictures

Atom light interaction

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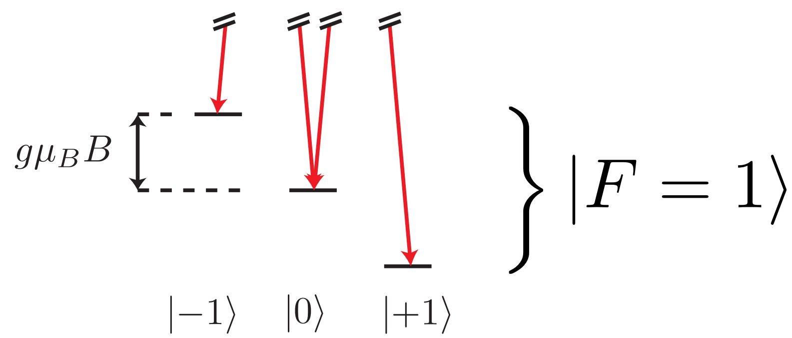

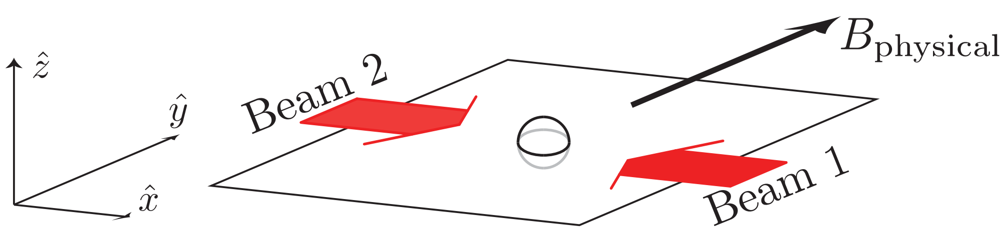

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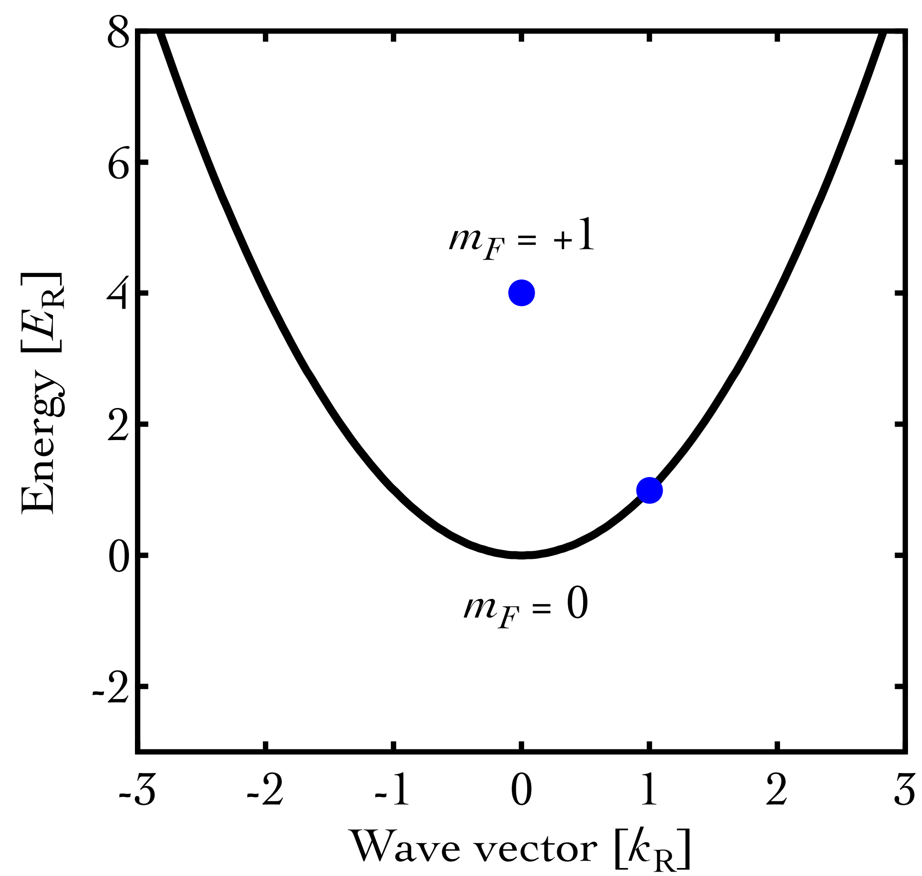
Coupled States



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Atom light interaction: pictures

Atom light interaction

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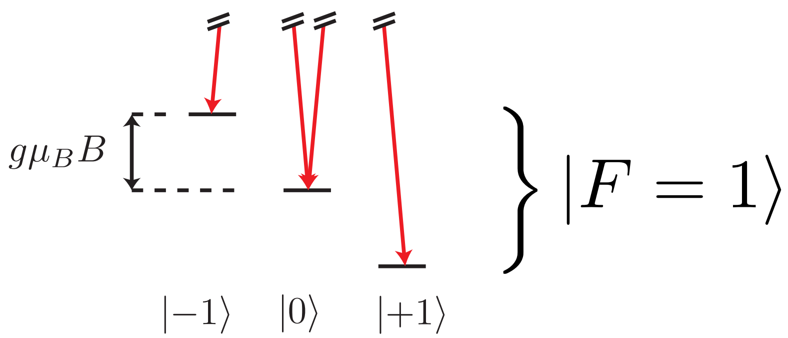

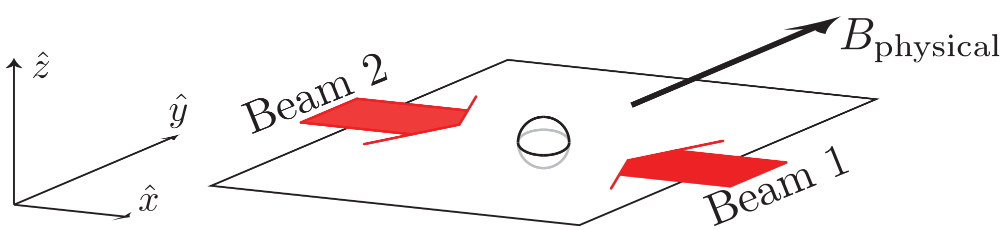

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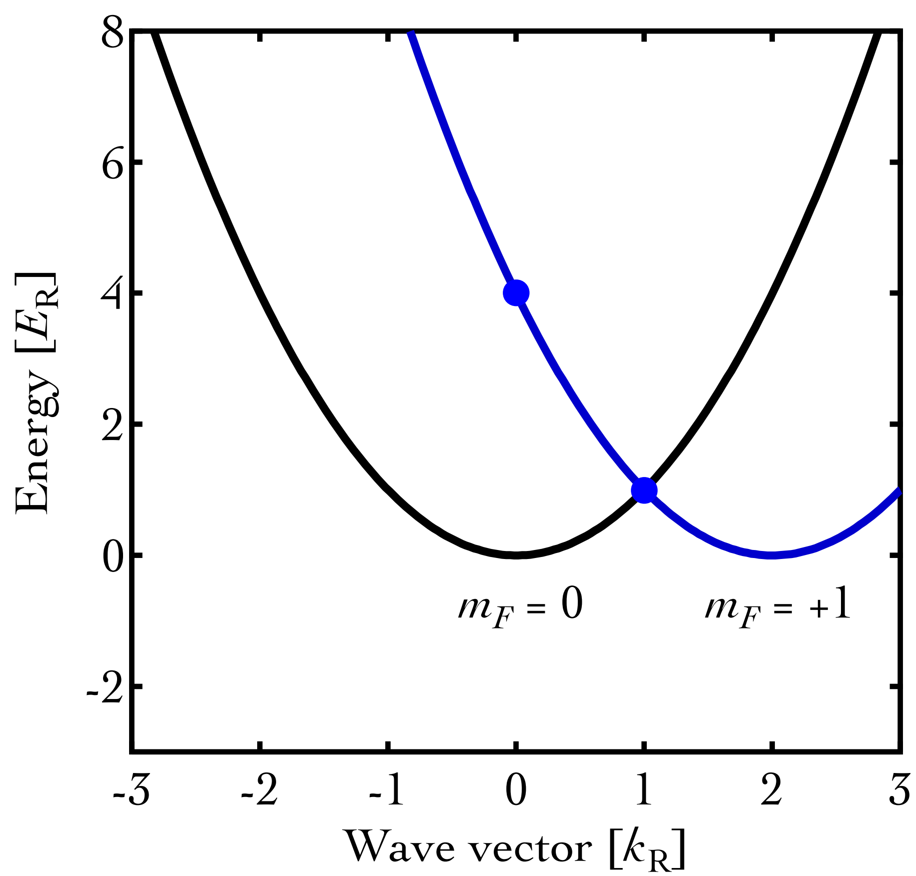
Coupled States



References

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Atom light interaction: pictures

Atom light interaction

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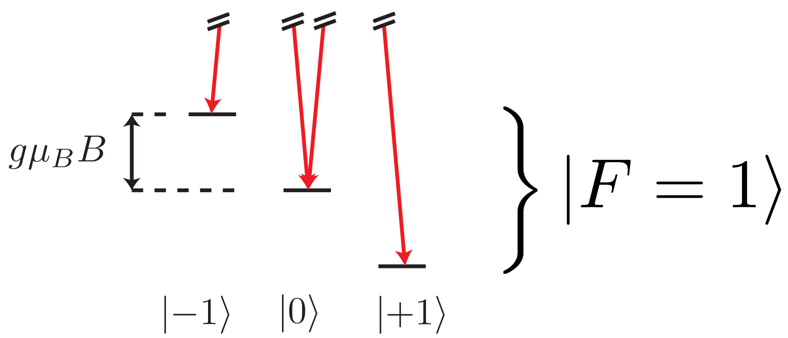

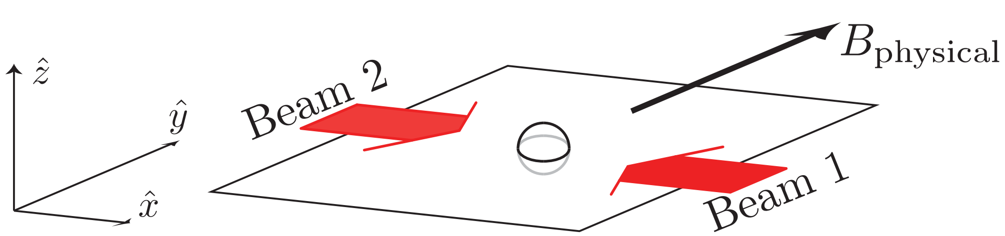

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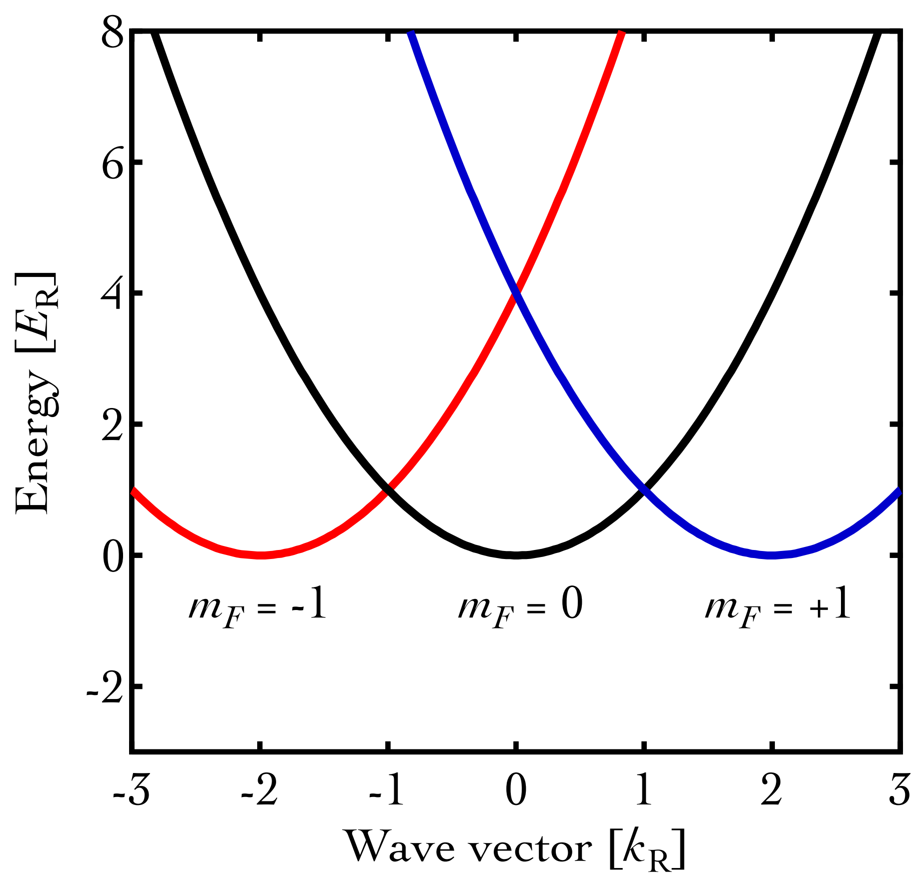

References

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[2] S.-L. Zhu, et al., PRL 240401 **97** (2006)
[3] Günter et al, PRA **79** 011604 (2009)
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Atom light interaction: pictures

Atom light interaction

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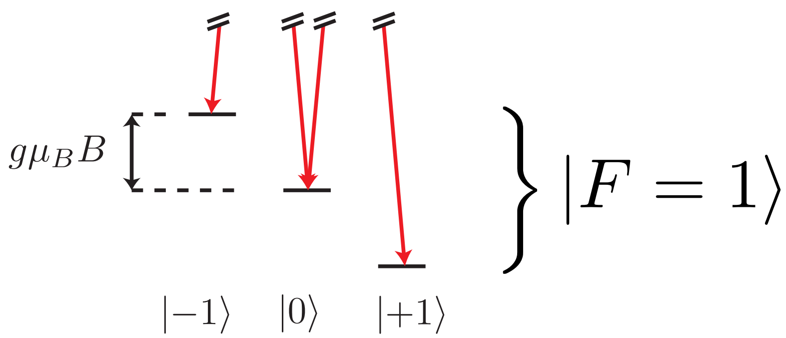

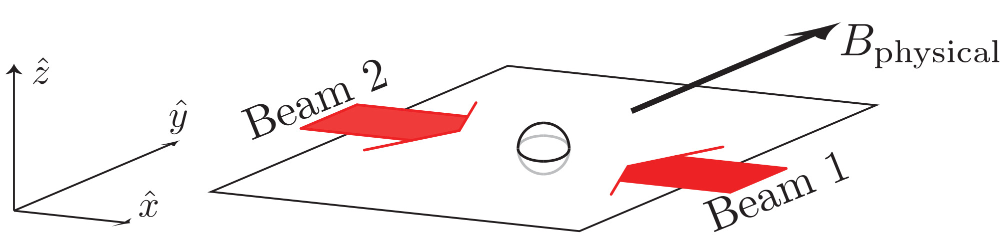

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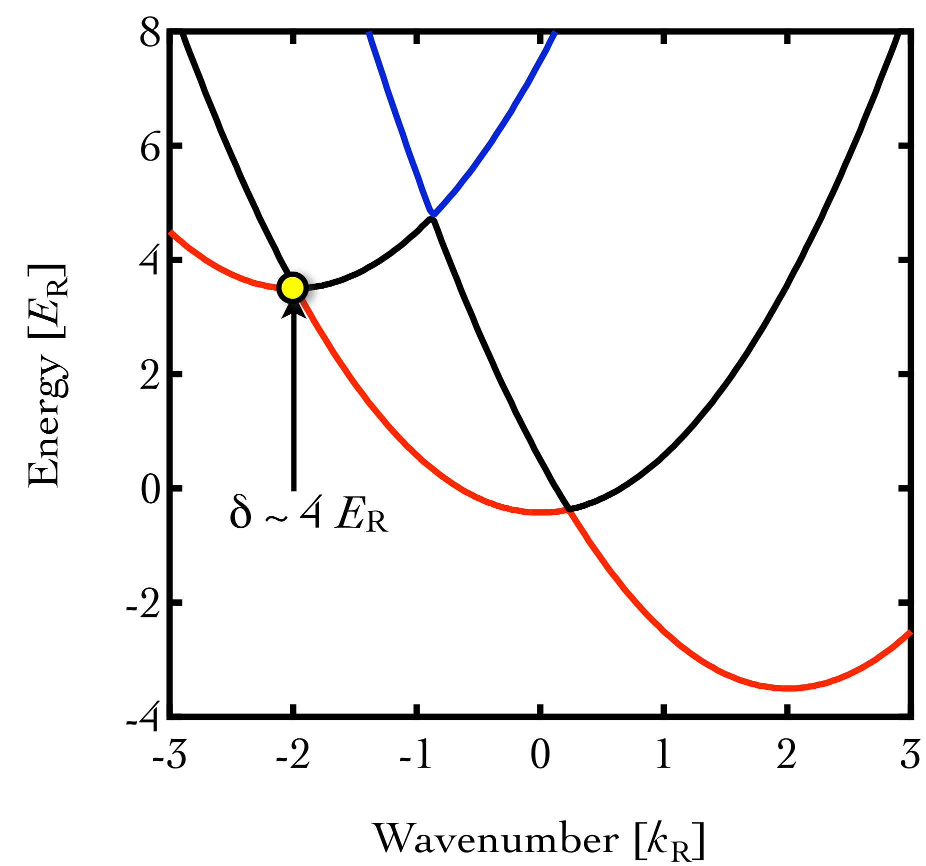
$$E_R \approx h \times 3 \text{ kHz} = k_{\rm B} \times 140 \text{ nK}$$

Coupled States

States will be labeled by:
(1) the “band index” and by
(2) a quasi-momentum k



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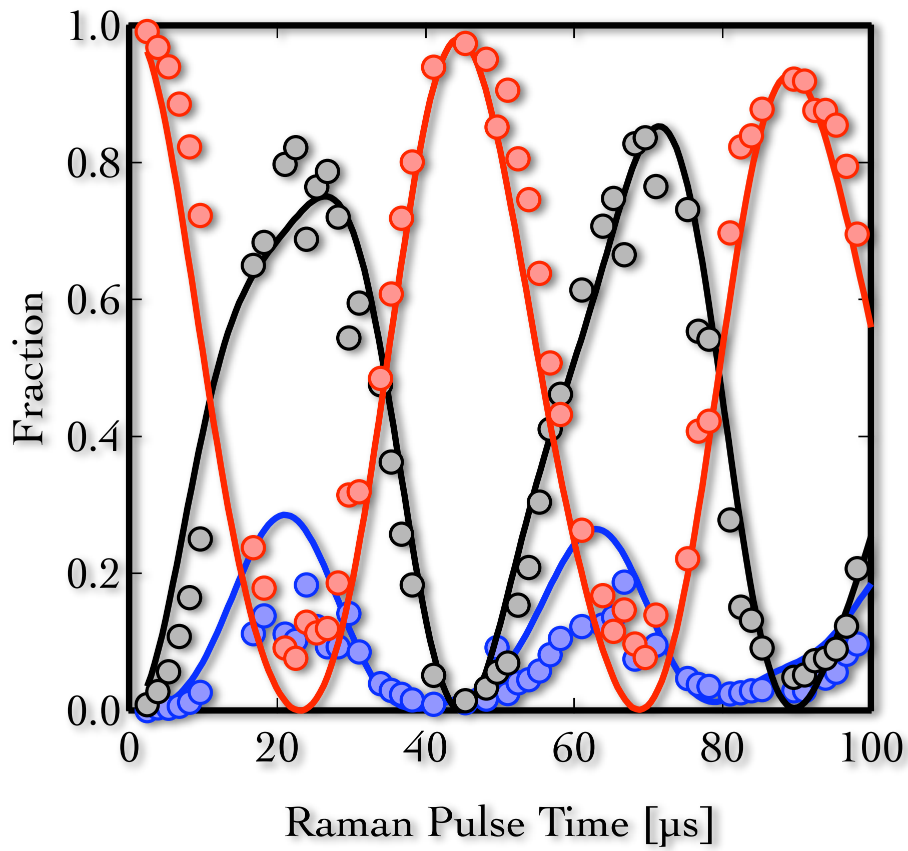
[2] S.-L. Zhu, et al., PRL 240401 **97** (2006)
[3] Günter et al, PRA **79** 011604 (2009)
[4] IBS, PRA 063613 **79** (2009)

Atom light interaction: pictures

Time evolution

In the sudden limit (Raman-Nath)
Population oscillations yield coupling



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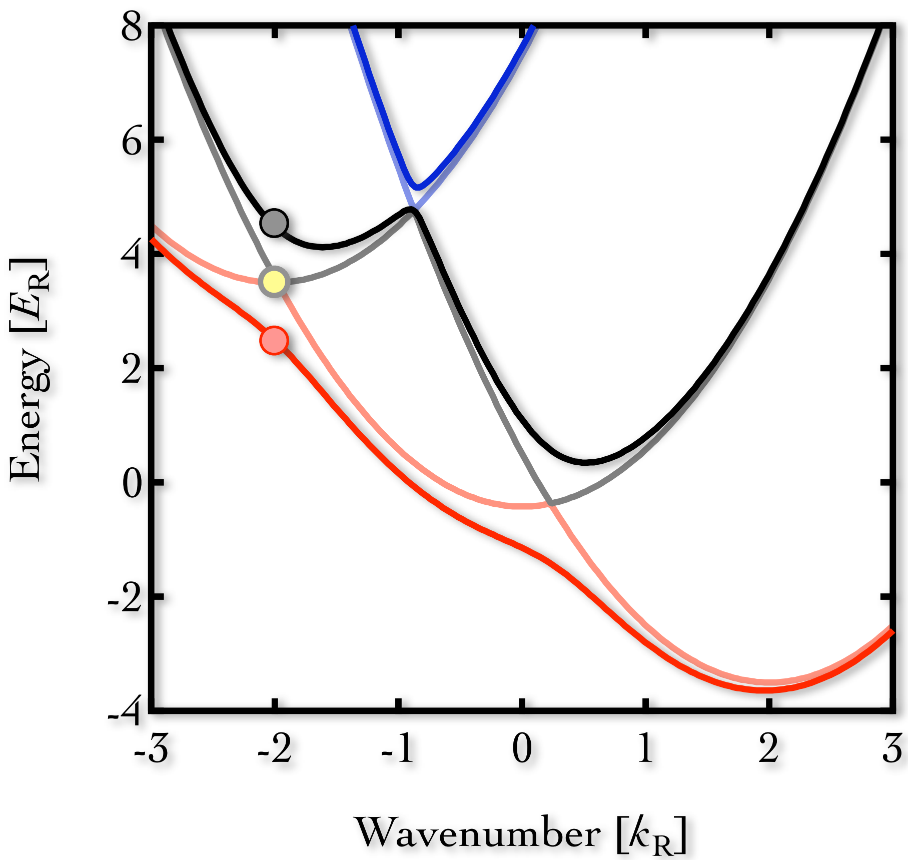
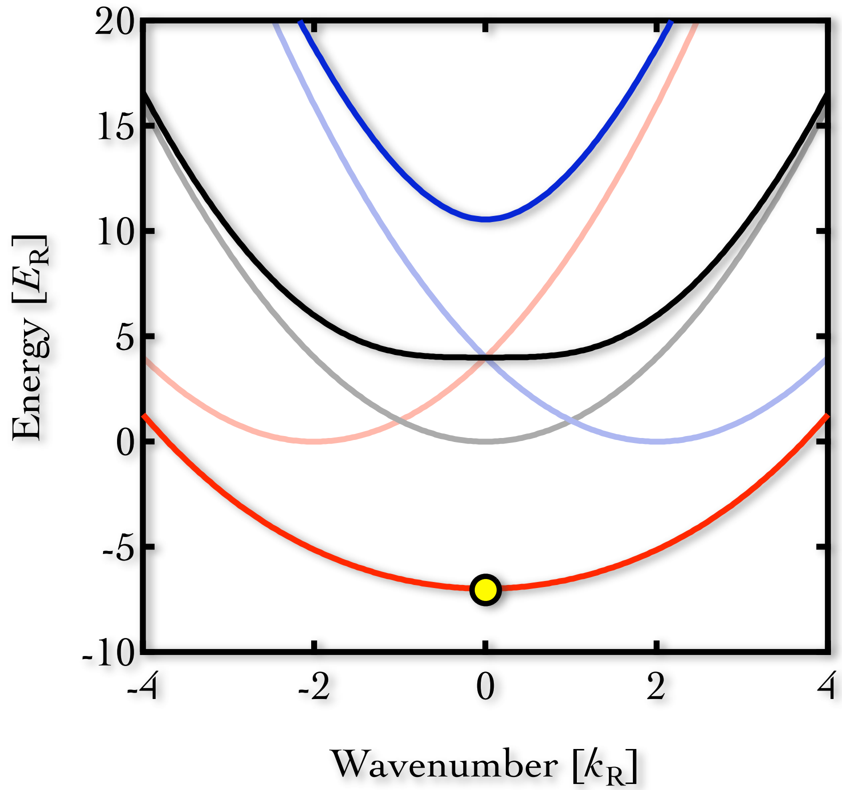
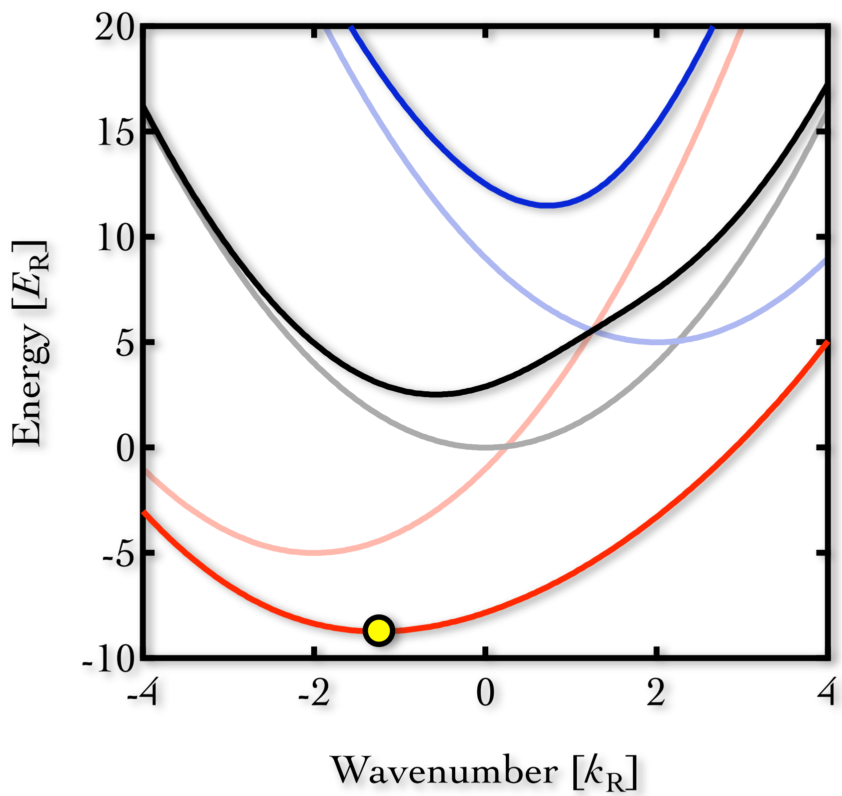


Y.-J. Lin et al, PRL **102** 130401 (2009)

Fundamental intuition



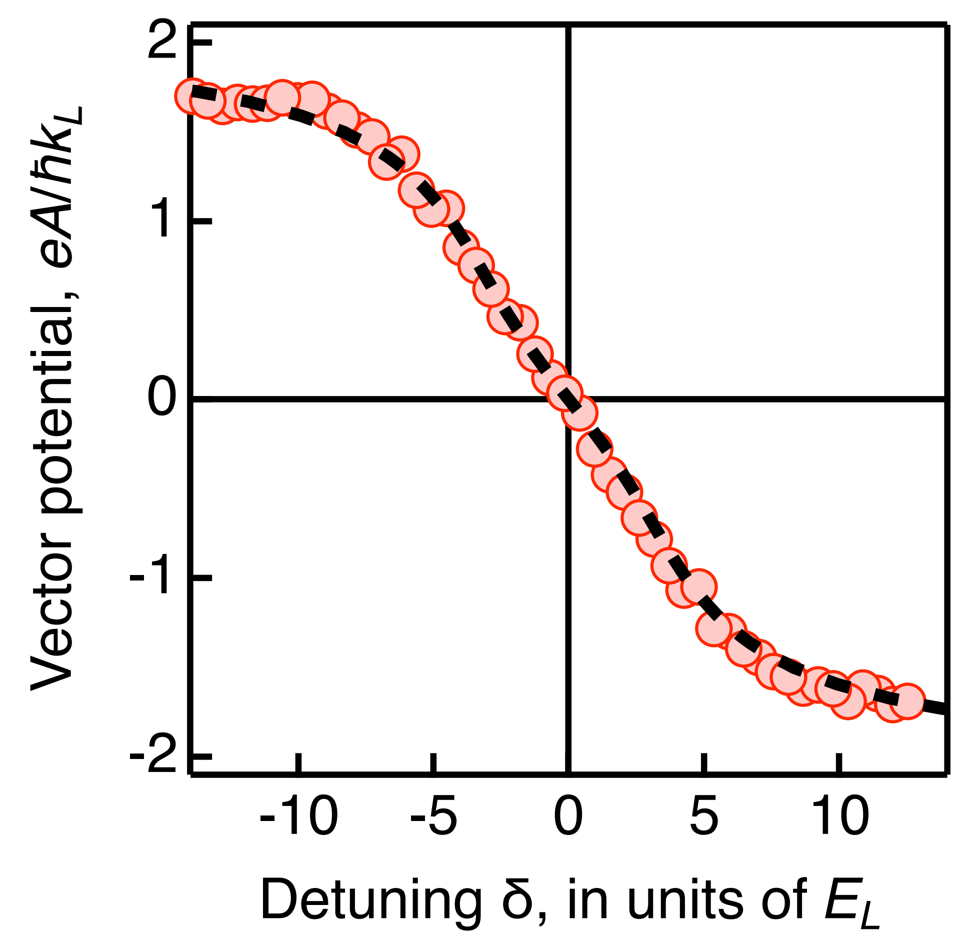


Transfer function

$$\hat{H} = \frac{\hbar^2}{2m}\left\{\left[k_x - \frac{qA_x(\delta,\Omega)}{\hbar}\right]^2 + k_y^2\right\} + V(\mathbf{x})$$

where $\delta(x,y,t)$ and $\Omega(x,y,t)$

The detuning and coupling specify the local synthetic vector potential



References

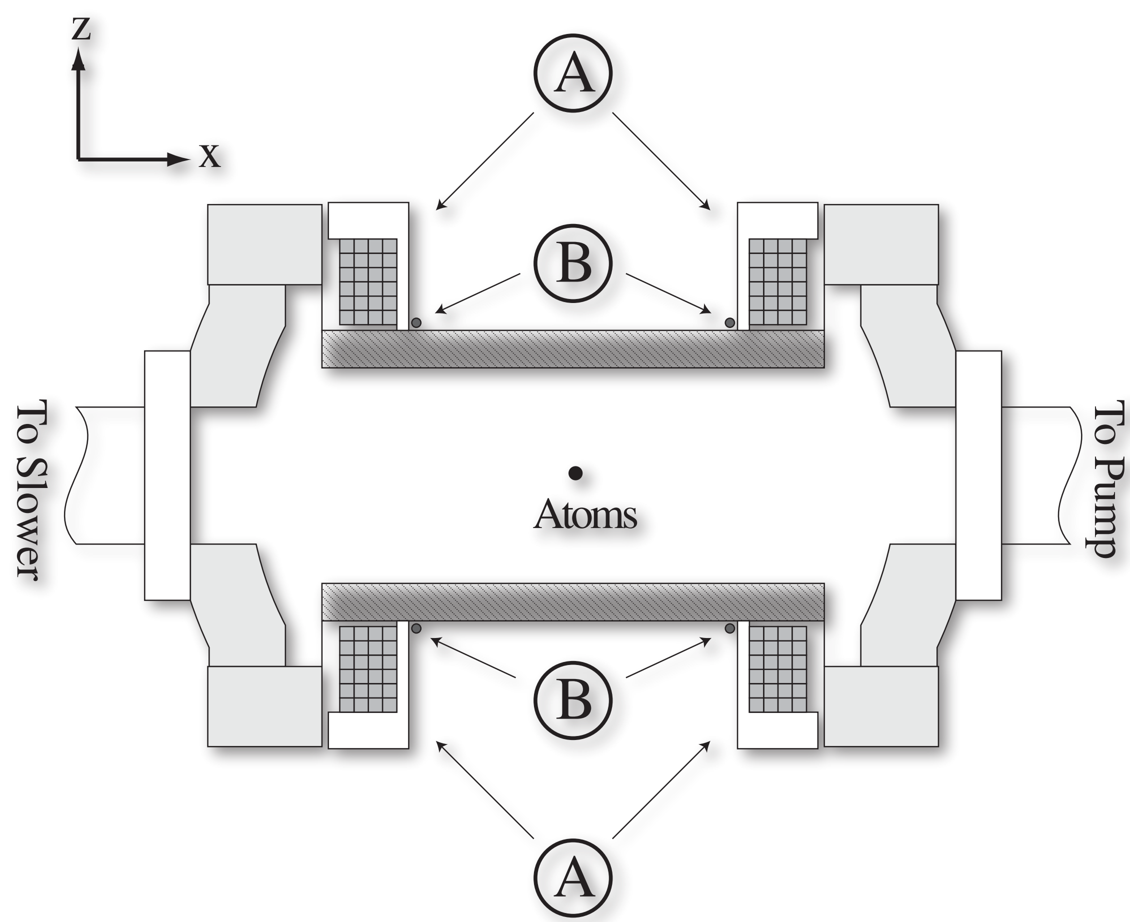
Y.-J. Lin et al, PRL (2009)

A laboratory tunable vector potential

Idea

We can control the *engineered* vector potential in time and space giving *synthetic* $\boldsymbol{E}$ and $\boldsymbol{B}$ fields.

Bias and quadrupole $\boldsymbol{B}$ fields = offset and gradient in detuning.



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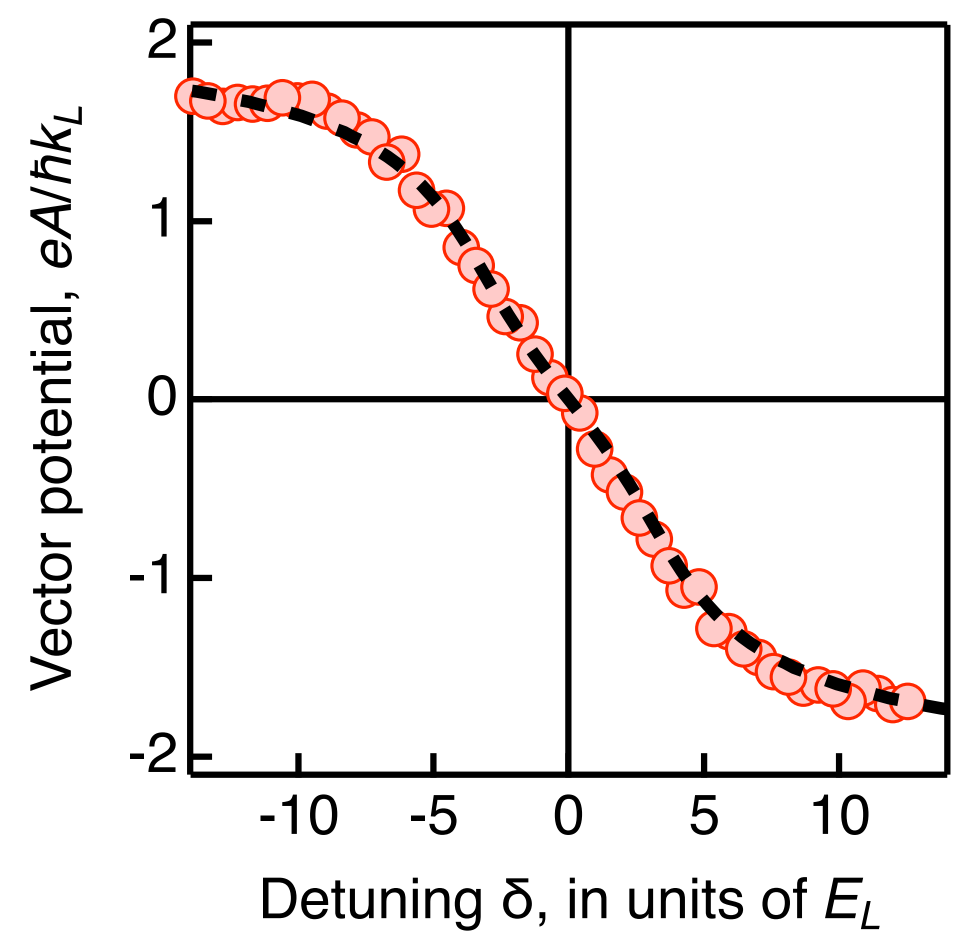


References

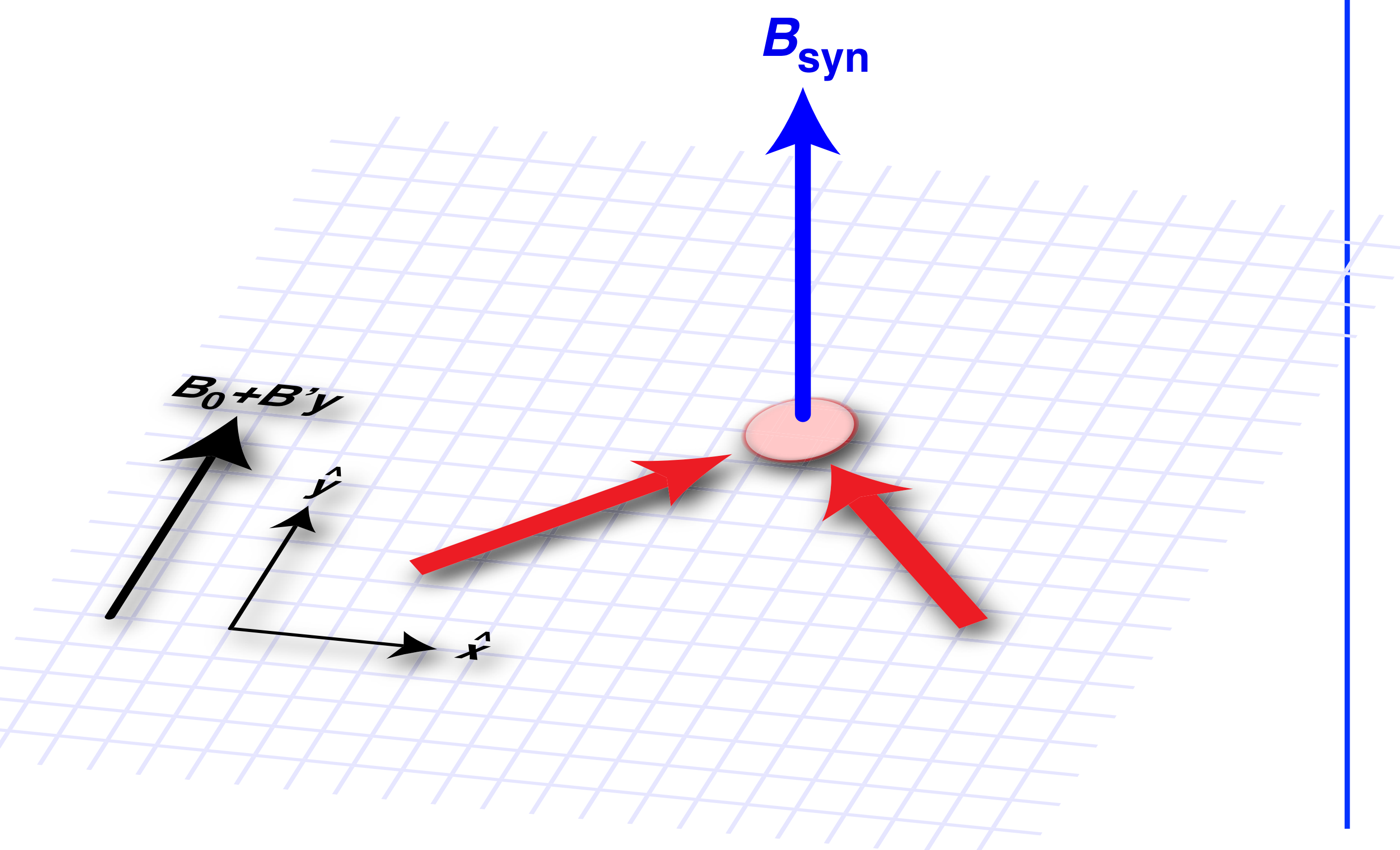
Y.-J. Lin et al, PRL (2009)

Synthetic magnetic field

A non-uniform vector potential

Spatial dependence gives magnetic fields and forces

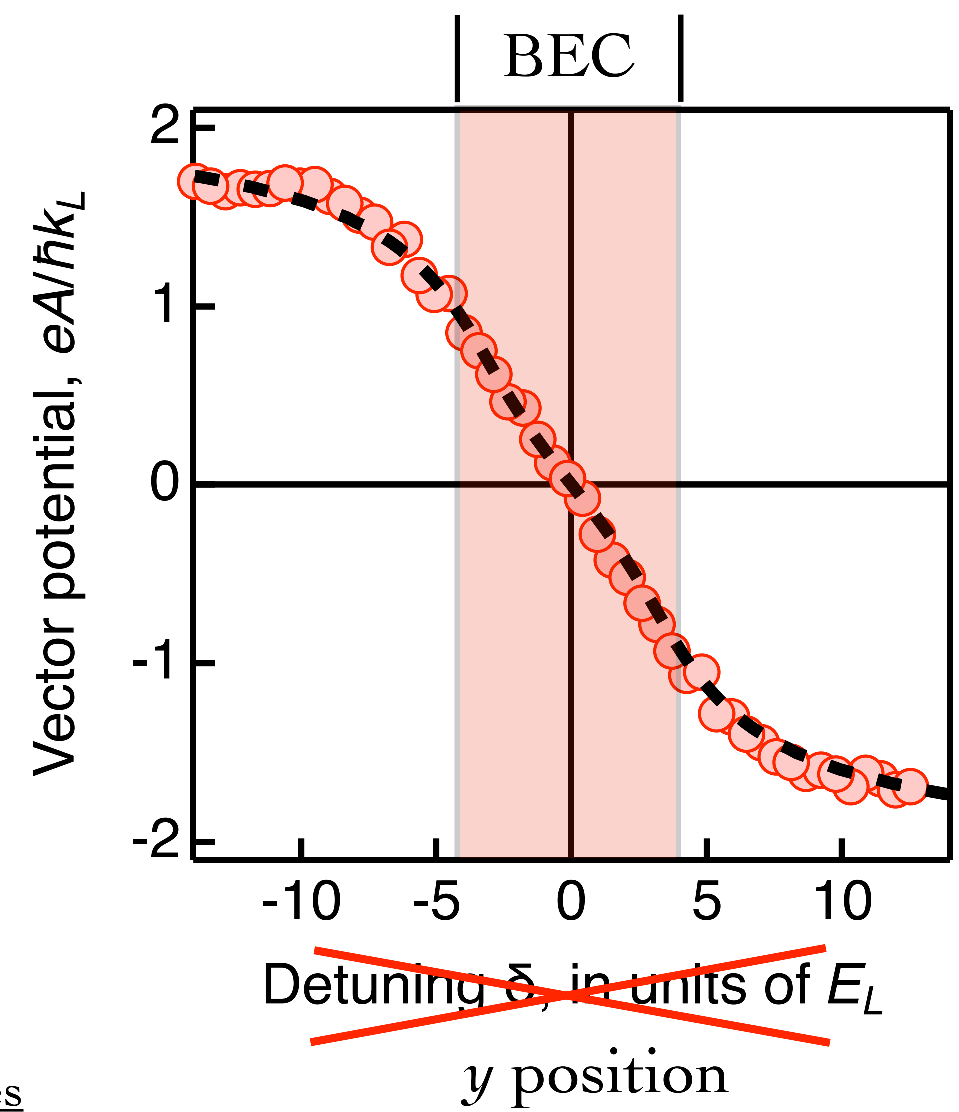
$$B = \nabla \times A$$



Transfer function

$$\hat{H} = \frac{\hbar^2}{2m}\left\{\left[k_x - \frac{qA_x(\delta,\Omega)}{\hbar}\right]^2 + k_y^2\right\} + V(\mathbf{x})$$

where $\delta(x,y,t)$ and $\Omega(x,y,t)$



References

Y.-J. Lin *et al*; Nature (2009)

Synthetic magnetic field

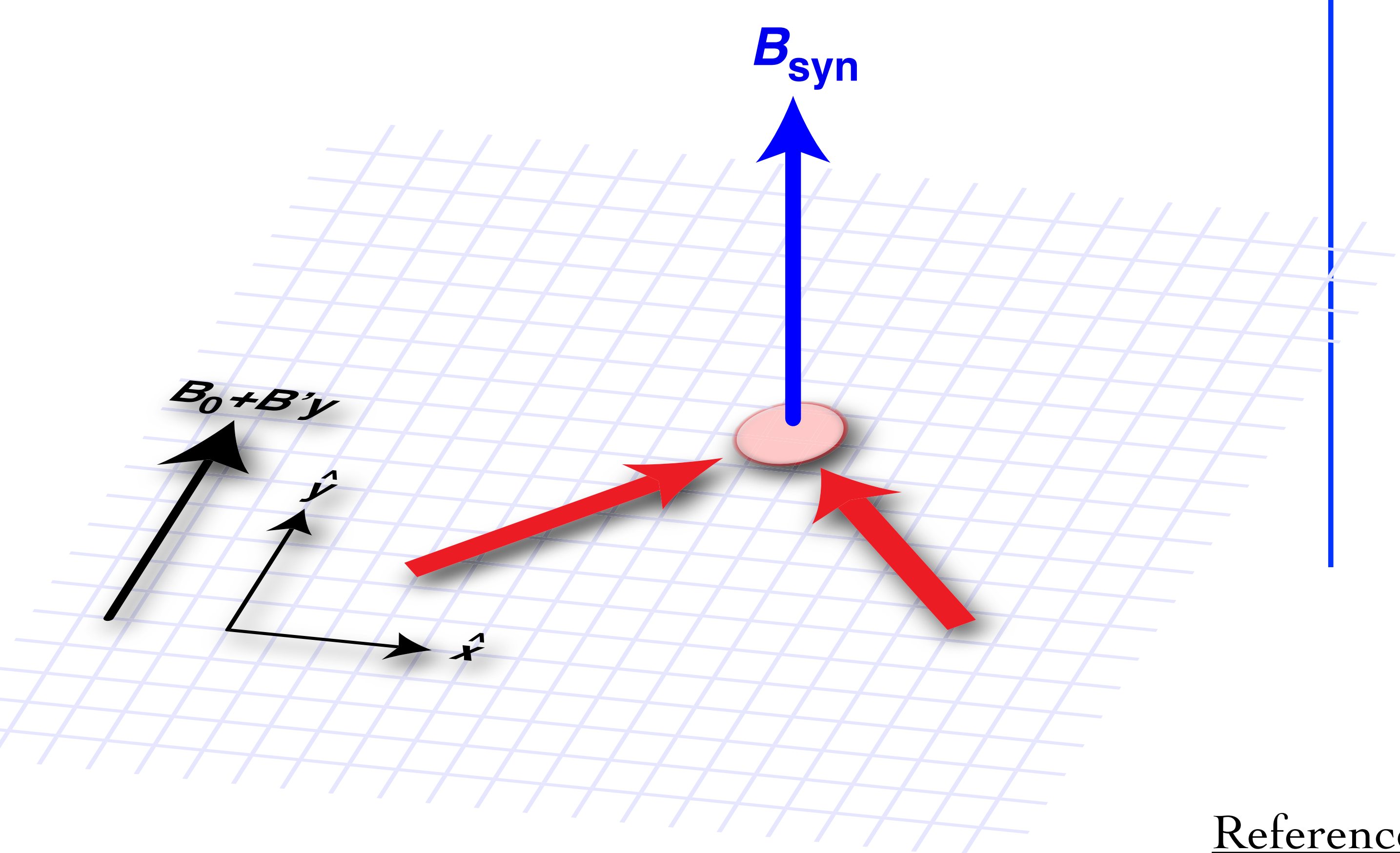
Loading procedure

Initial state
RF dressed state (RF on, ramp B to resonance)
Raman + RF dressed state (Ramp Raman on)
Raman only dressed state (Ramp RF off)

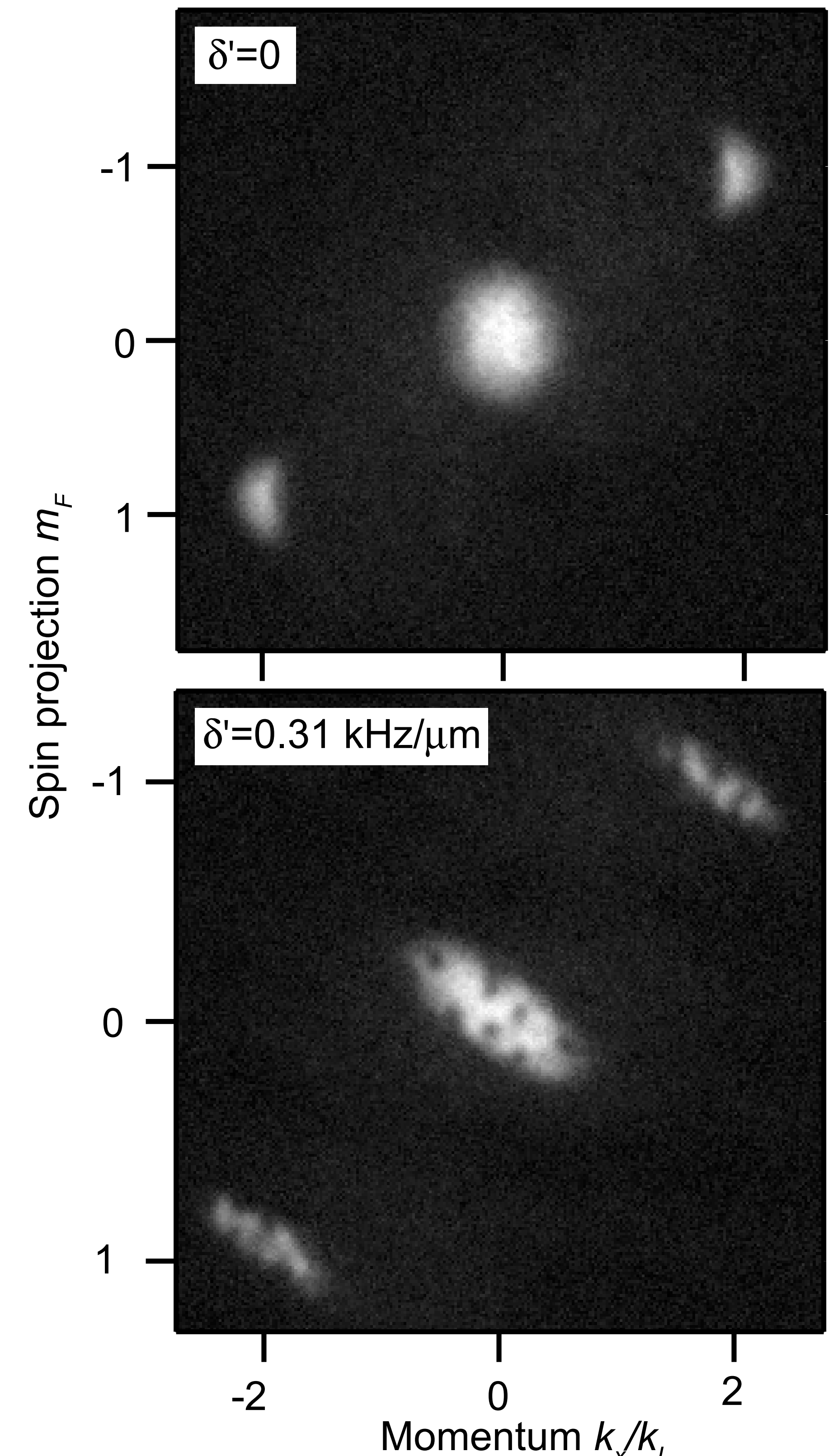
Ramp field gradient on (from 0 to 500 Hz/μm)

Equilibrate for 500 ms

TOF imaging



Outcome

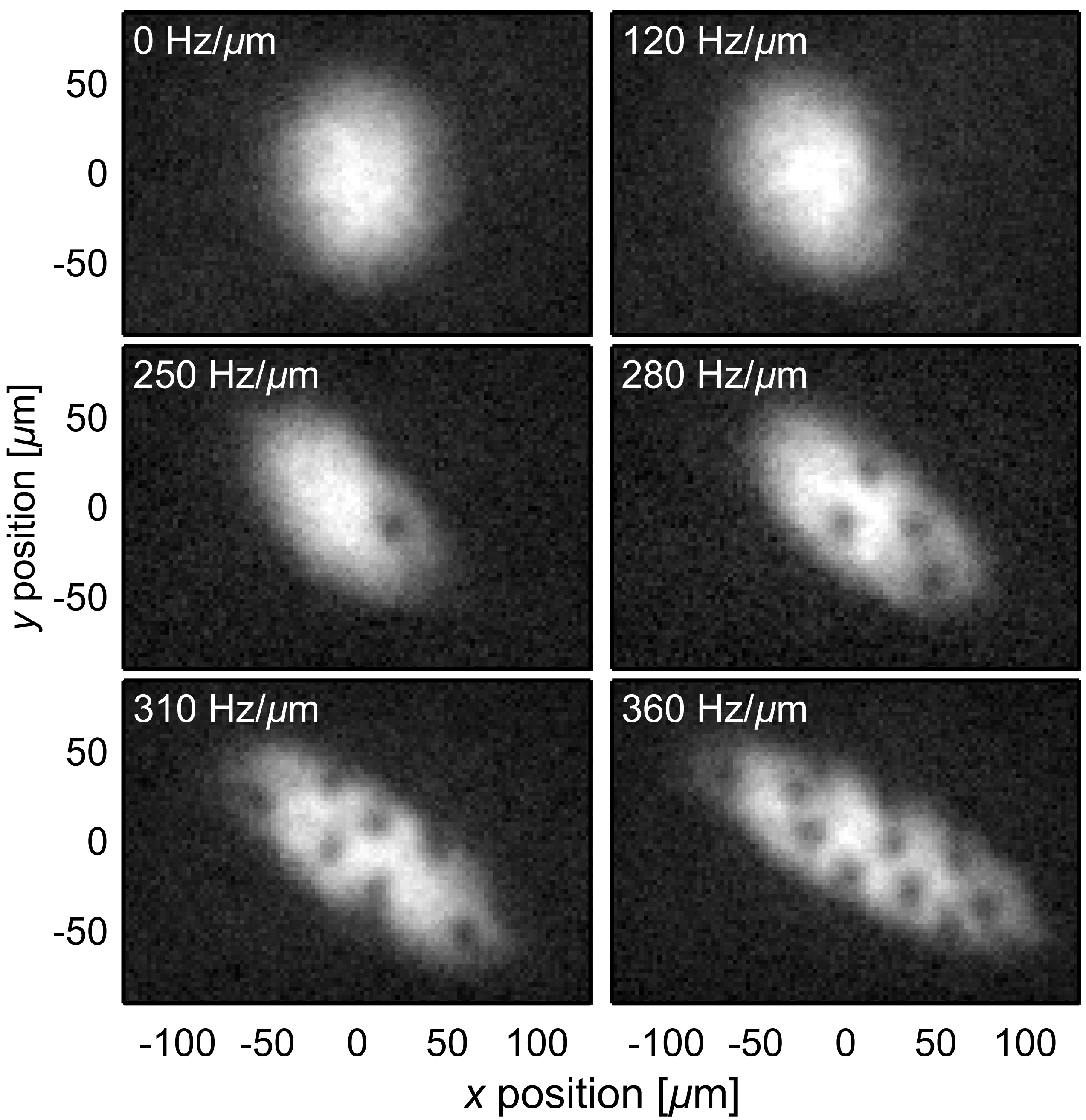


References
Y.-J. Lin *et al* Nature (2009)

Expected properties of BEC with fields

Vortex number

Spatial dependence gives magnetic fields and forces

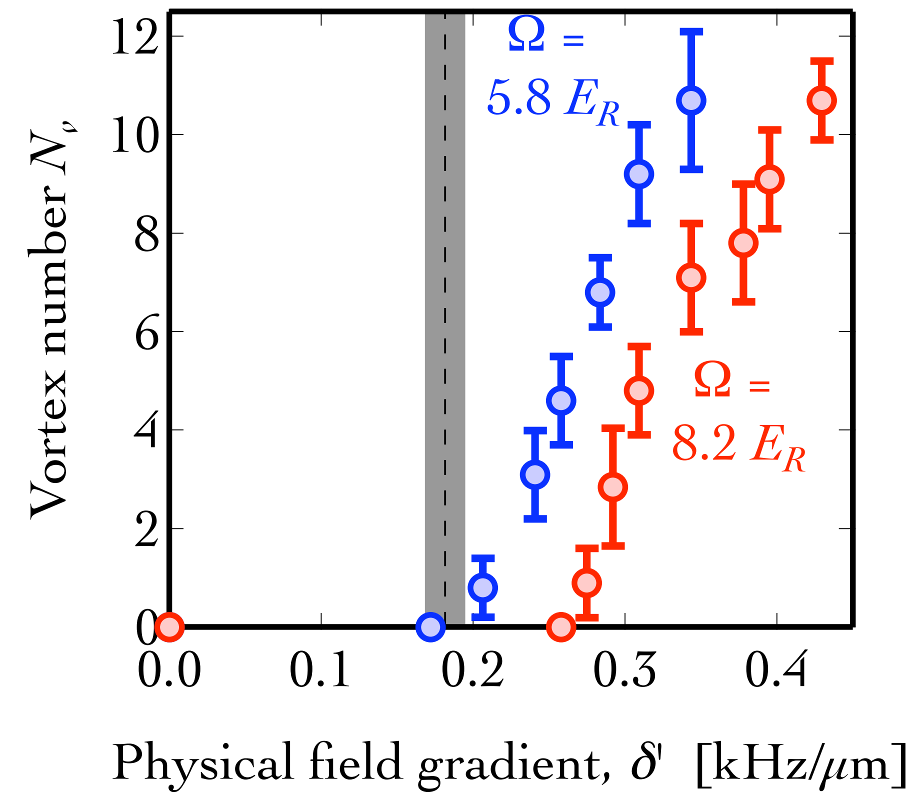


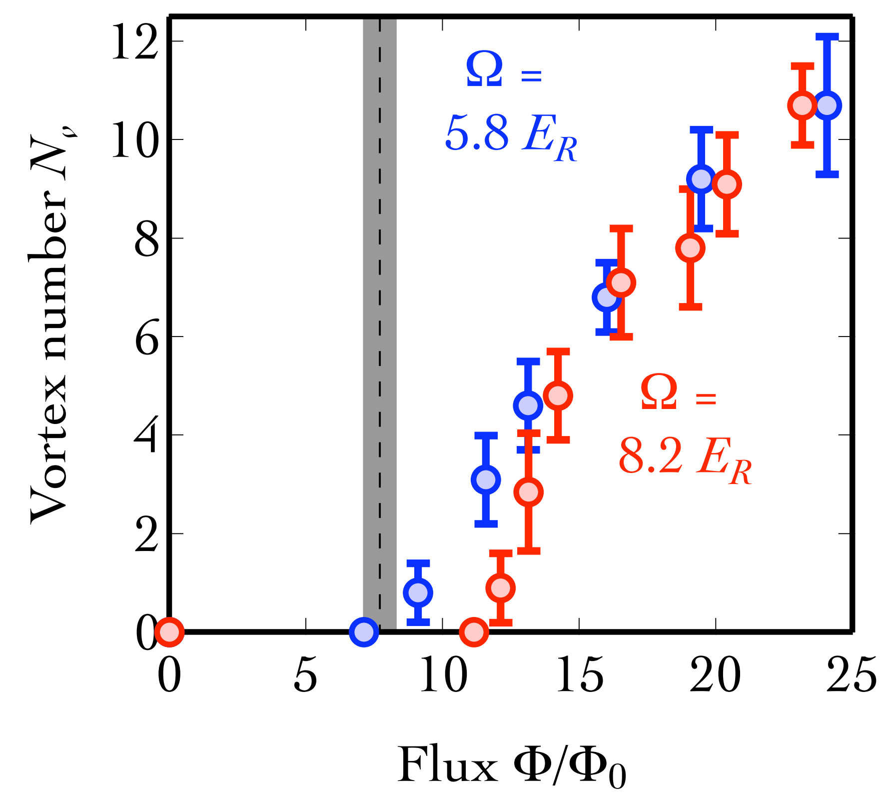
References

Y.-J. Lin *et al*; Nature (2009)

Extensive review: A.L. Fetter, RMP **81** 647 (2009)

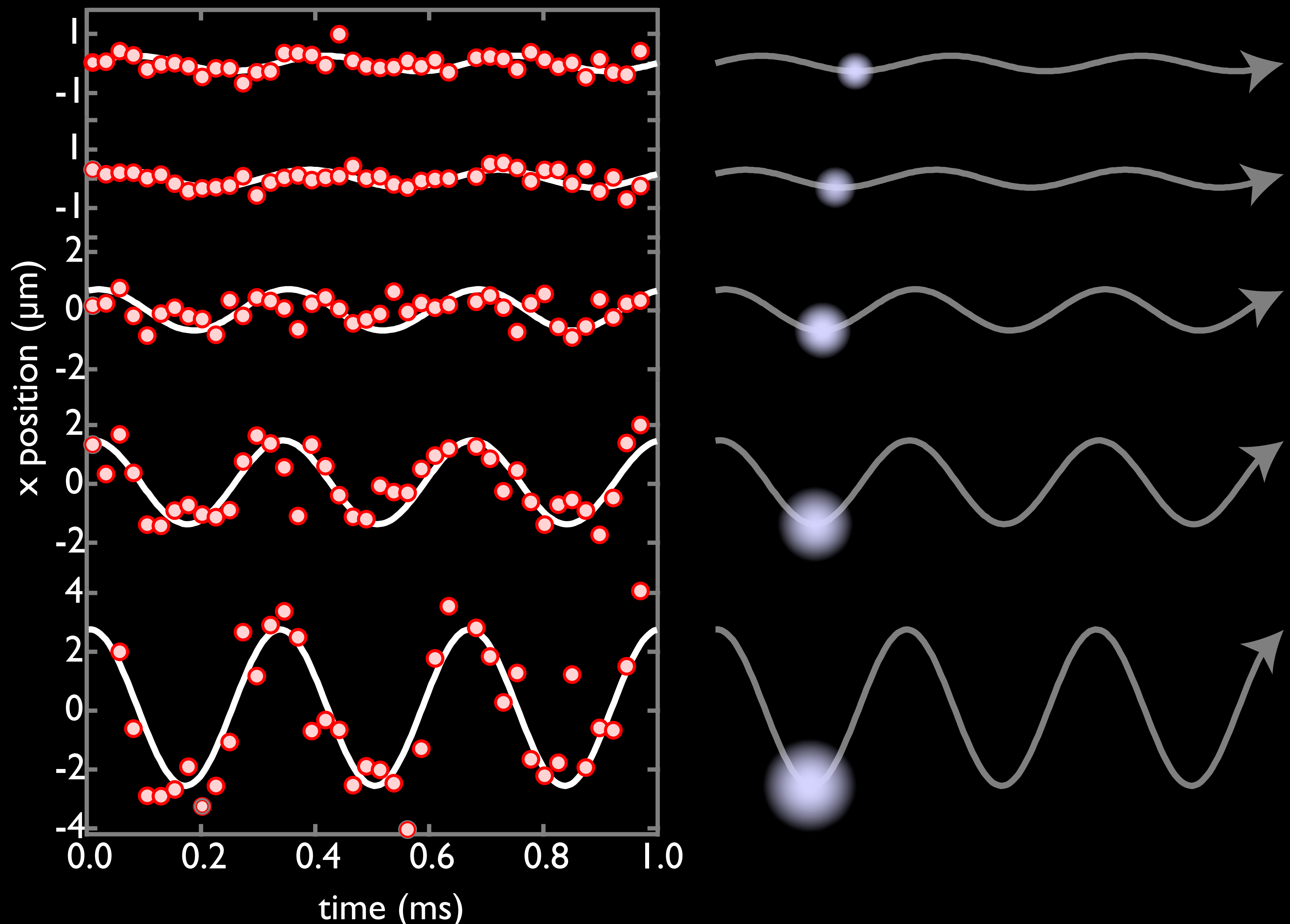
Critical field for vortex formation





Atomic Zitterbewegung

$$\hat{H}_D = c\hat{p}_x\check{\sigma}_z + m_e c^2\check{\sigma}_x$$

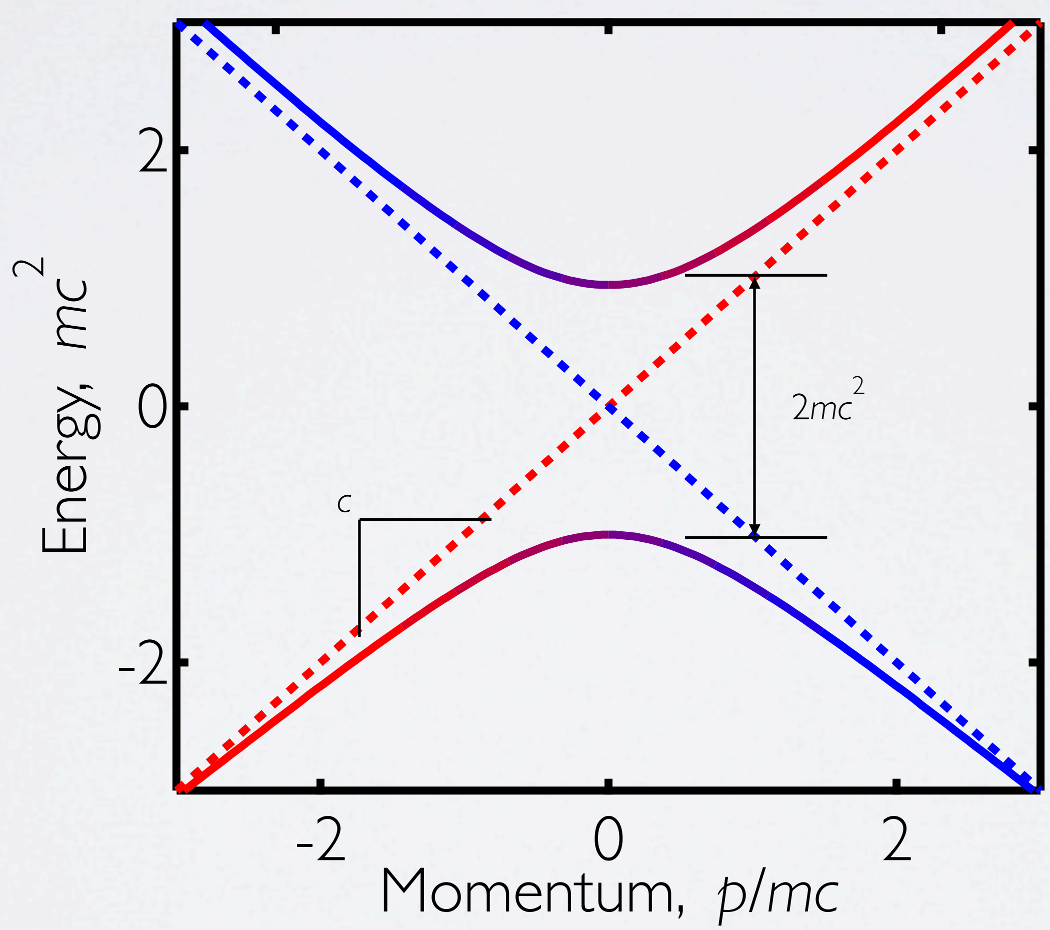


ZITTERBEWEGUNG: "JITTERING" MOTION

$$f = \frac{2mc^2}{h} = 2.5 \times 10^{20}\ \text{Hz}$$

$$\delta x = \pm\frac{\lambda_C}{4\pi} = 2.4\ \text{pm}$$

$$\hat{H}_{\mathrm{D}}\psi = \left(c\hat{p}_x\check{\sigma}_z + m_e c^2\check{\sigma}_x\right)\psi,$$



Refs.

Schrodinger (1930)
David and Cserti PRB (2010)

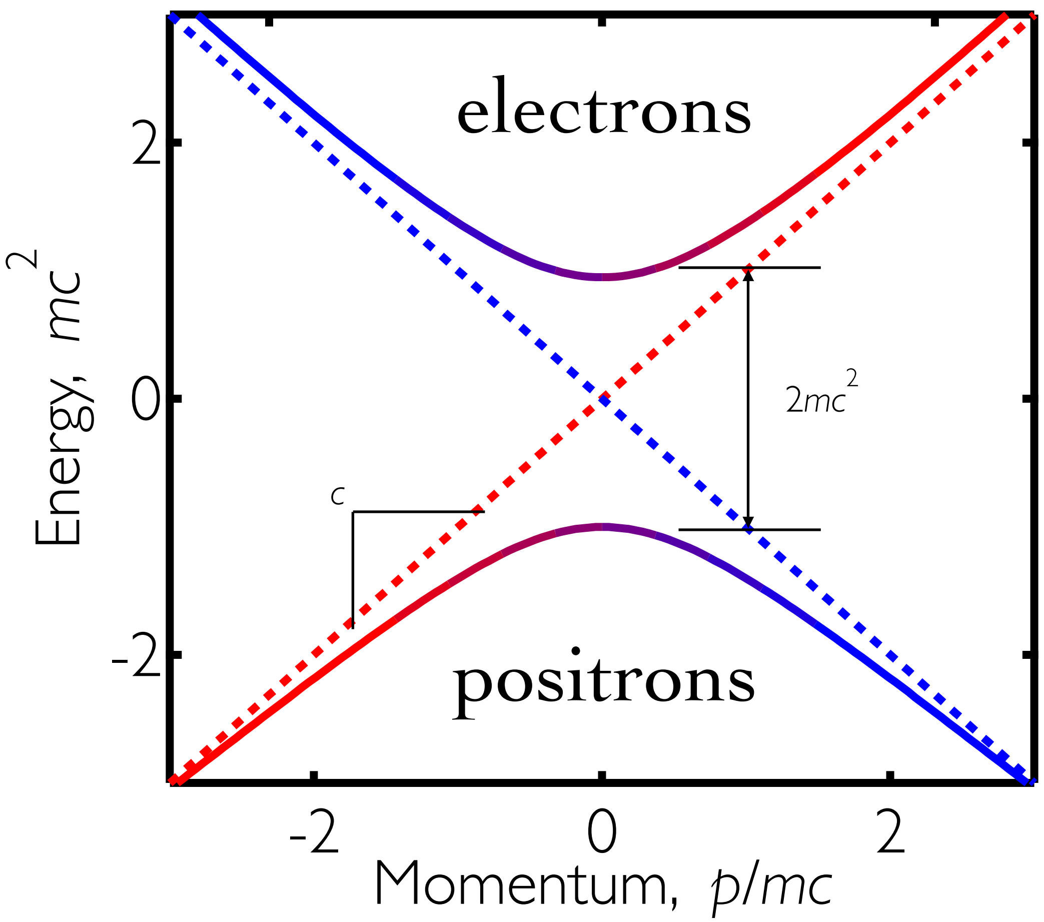
Zitterbewegung

Expected tiny "jittering" of electrons (small and very fast)

$$f = \frac{2mc^2}{h} = 2.5 \times 10^{20}\ \text{Hz}$$

$$\delta x = \pm \frac{\lambda_C}{4\pi} = 2.4\ \text{pm}$$

$$\hat{H}_{\mathrm{D}}\psi = \left(c\hat{p_x}\check{\sigma}_z + m_e c^2 \check{\sigma}_x\right)\psi,$$



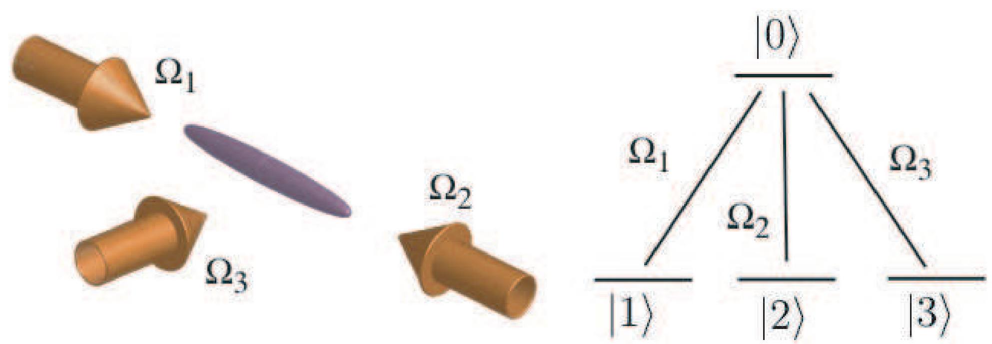
$$\frac{dx}{dy} = \frac{1}{i\hbar}[\hat{x}, \hat{H}] = c\check{\sigma}_z = v$$

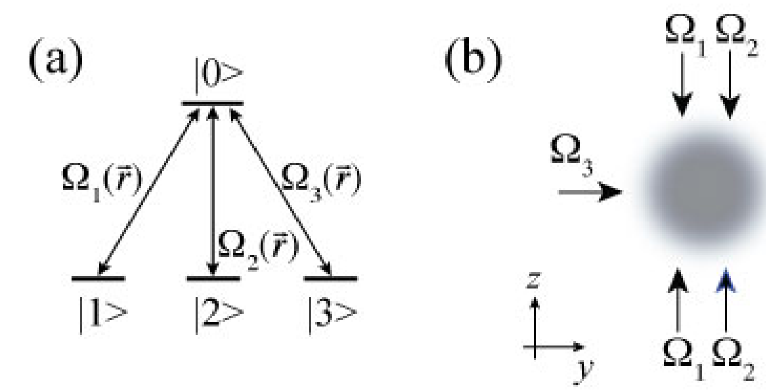
$$\frac{dv}{dt} = \frac{2mc^3}{\hbar}\check{\sigma}_y$$

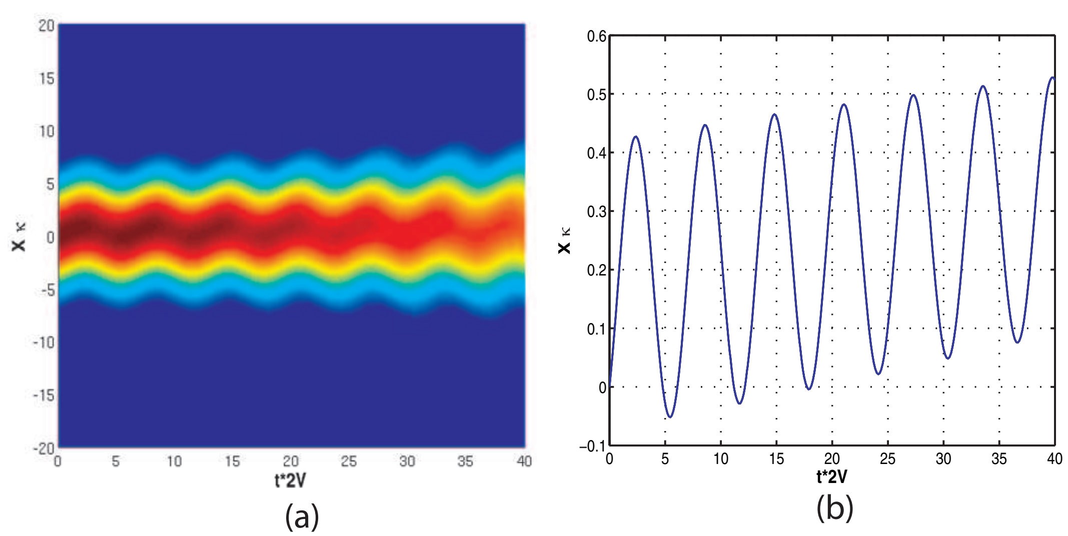
$$\frac{d^2v}{dt^2} = \frac{4mc^4}{\hbar}\check{\sigma}_x\hat{k} - \left(\frac{2mc^2}{\hbar}\right)^2 v$$

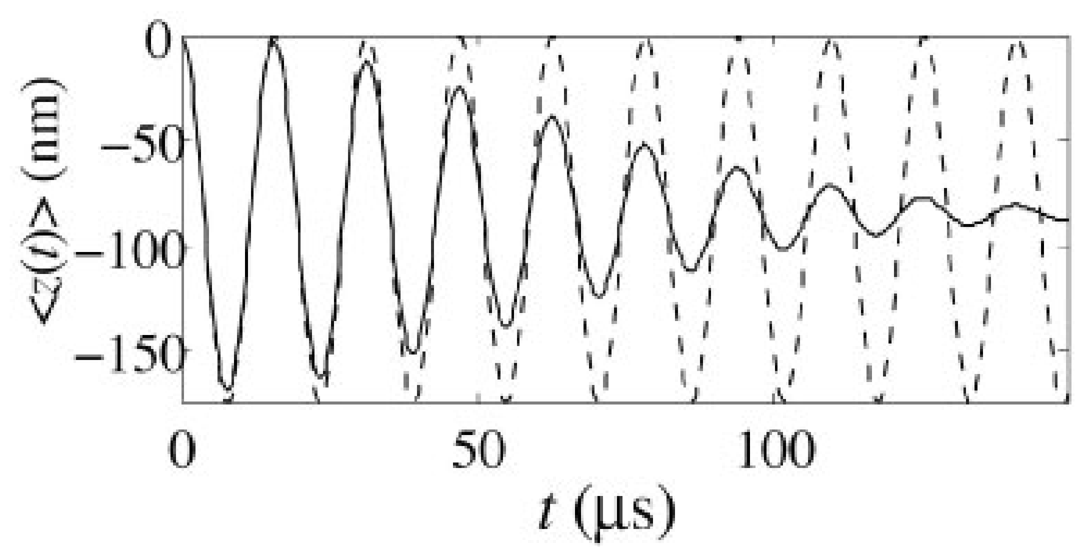
Ref.
David and Cserti PRB (2010)

Proposals with cold atoms







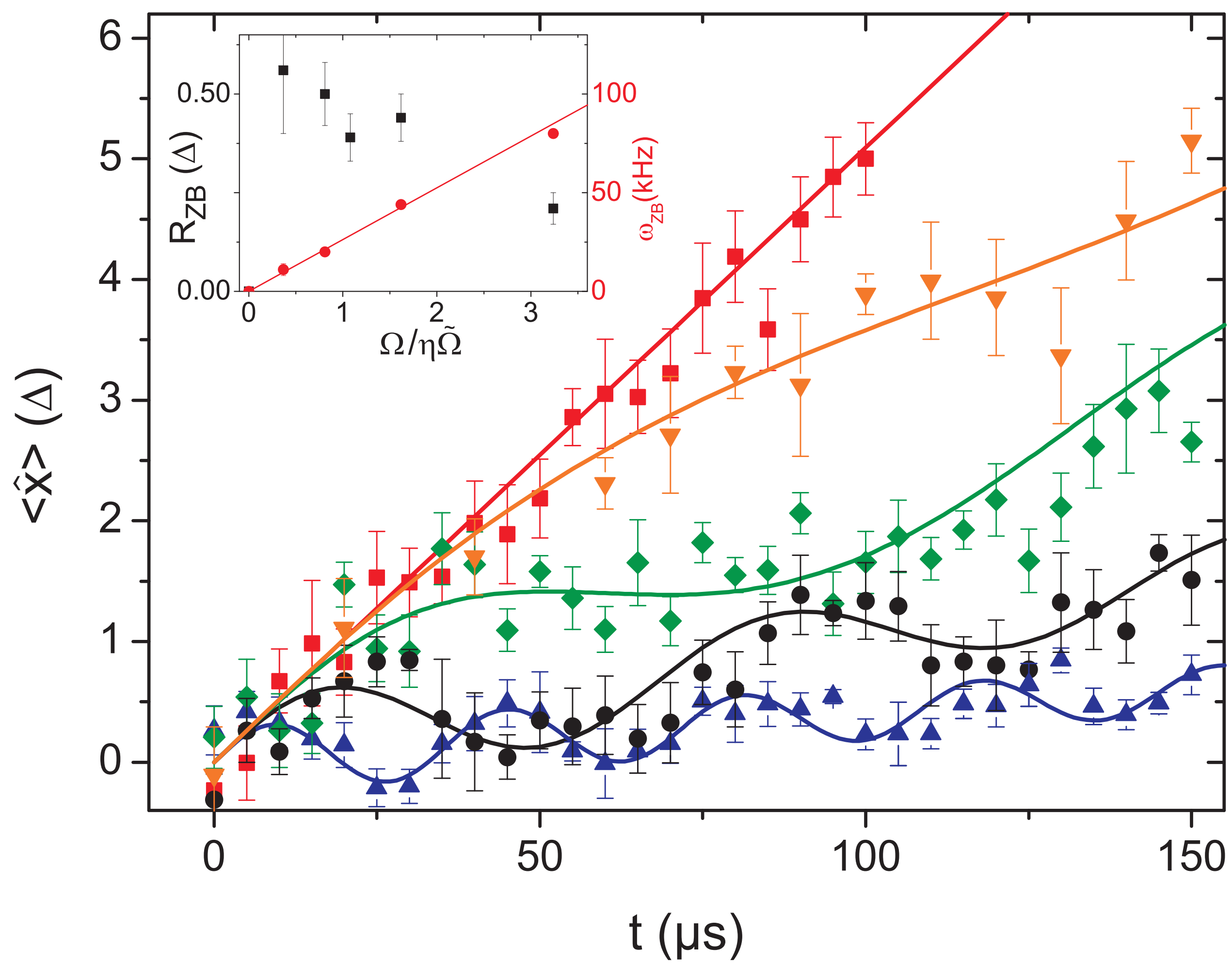


<u>Ref.</u>

Vaishnav and Clark PRL (2008)
Merkl, Zimmer, Juzeliūnas, and Ohberg EPL (2008)
Zhang, Gong, and C. H. Oh arXiv:1208.3005 (2012)

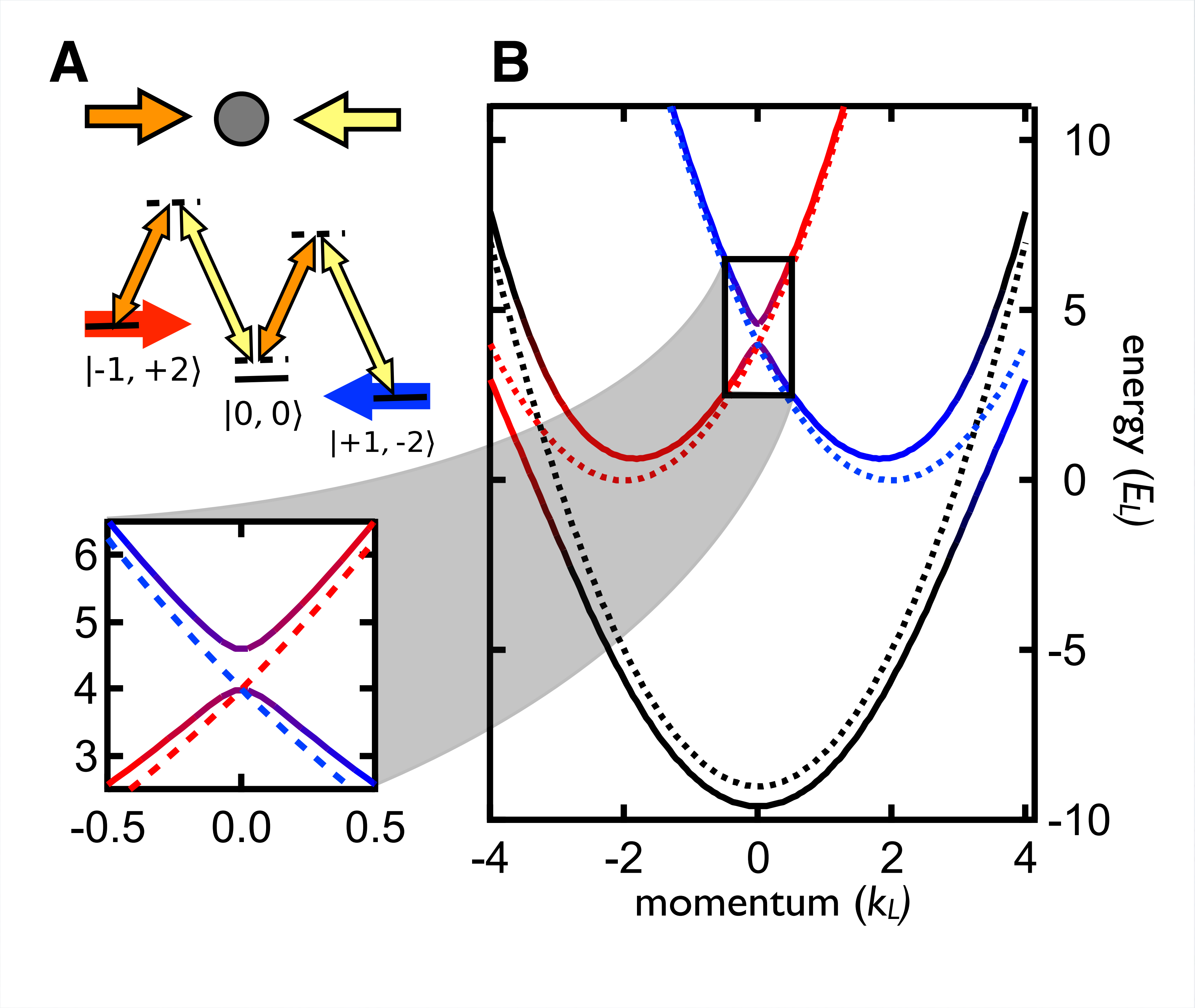
Analog realized with individual atomic ions

Measuring x and p quadratures in harmonic trap



Ref.
Gerritsma, et al. Nature (2010)

BEC EXPERIMENT



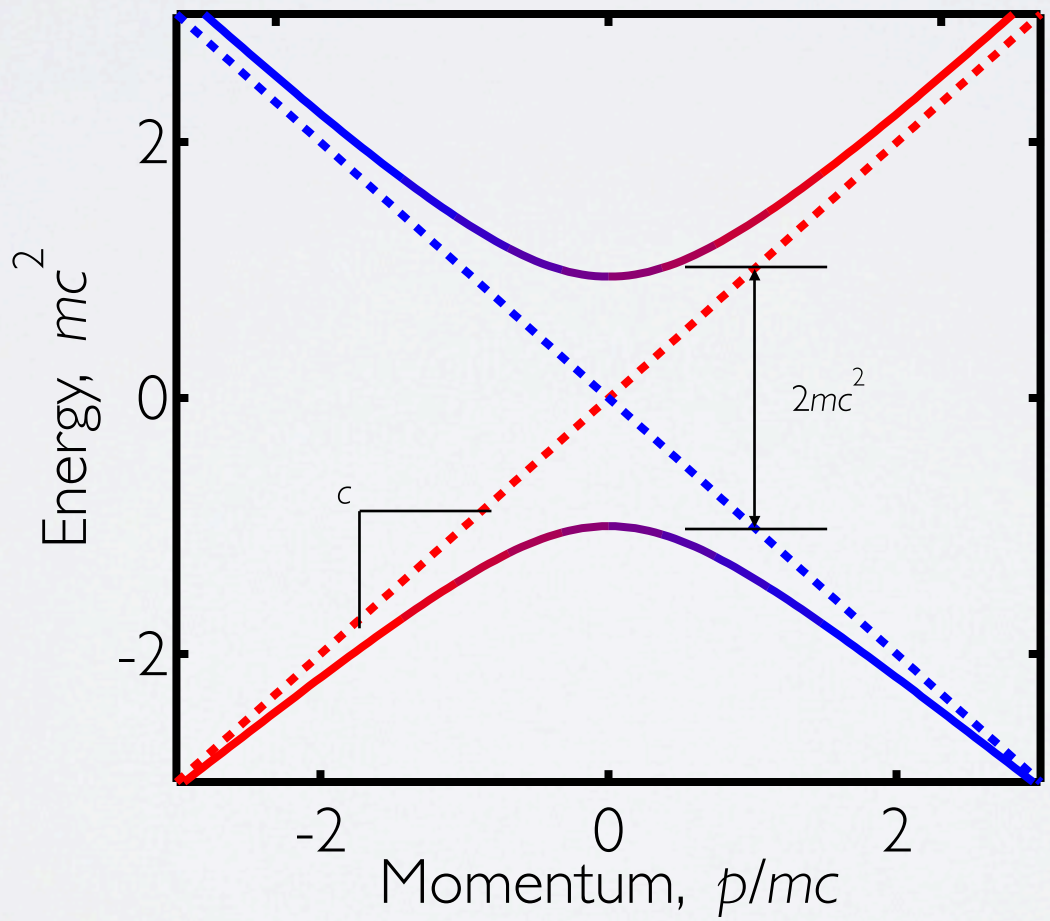
Theory Ref.
Zhang, Gong, and C. H. Oh arXiv:1208.3005 (2012)

ULTRASLOW RELATIVISTIC SYSTEM

$$2mc^2 = \hbar\Omega_2 \approx h \times 1\ \mathrm{kHz}$$

$$c = \hbar 2k_r/m_{\mathrm{Rb}} \approx 11\ \mathrm{mm/s}$$

$$\hat{H}_{\mathrm{D}}\psi = \left(c\hat{p_x}\check{\sigma}_z + m_e c^2 \check{\sigma}_x\right)\psi,$$



Ref.

Zhang, Gong, and C. H. Oh arXiv:1208.3005 (2012)

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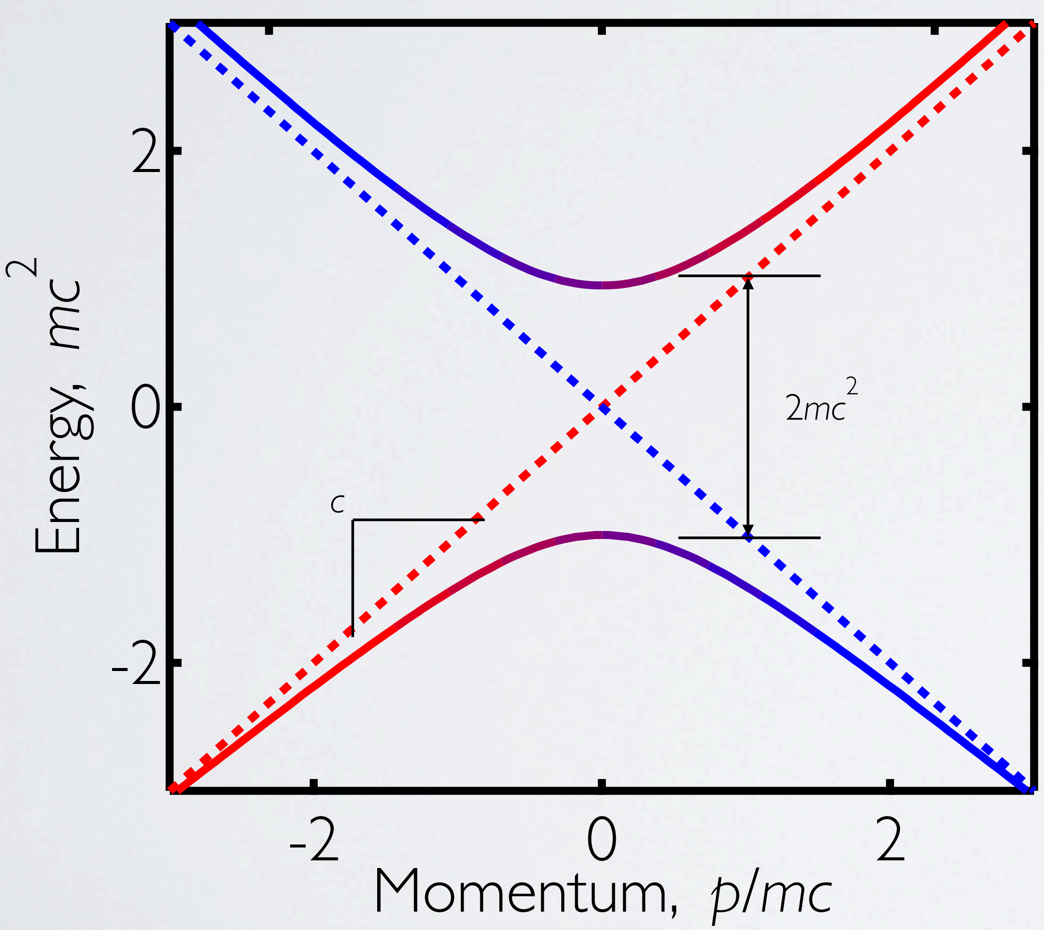
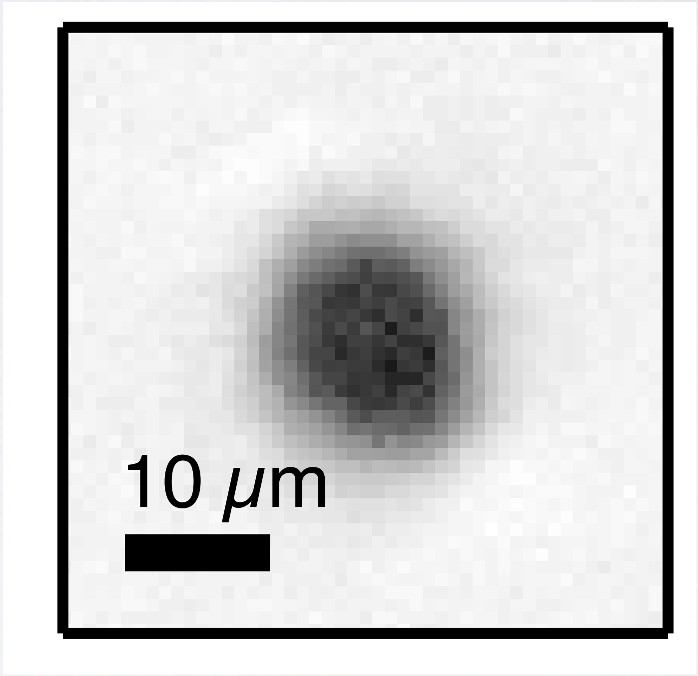


Measure real position



Ref.
Zhang, Gong, and C. H. Oh arXiv:1208.3005 (2012)

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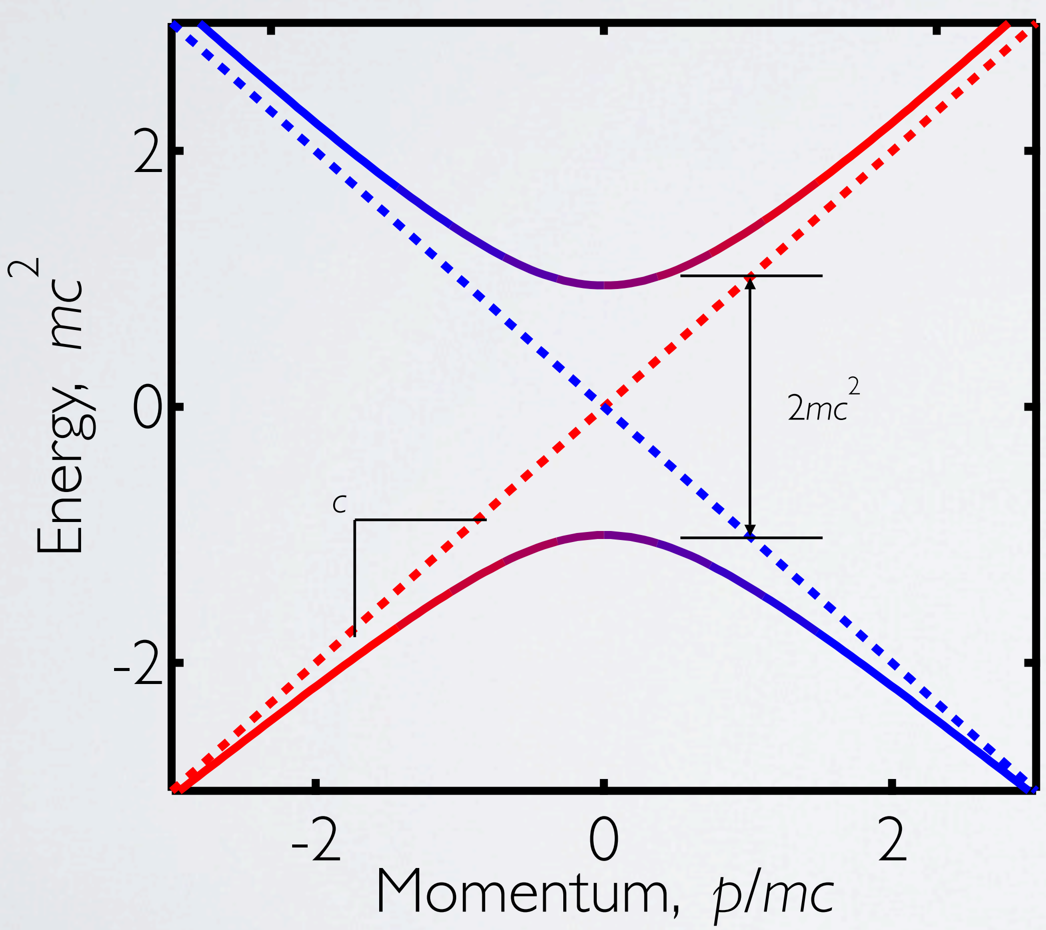
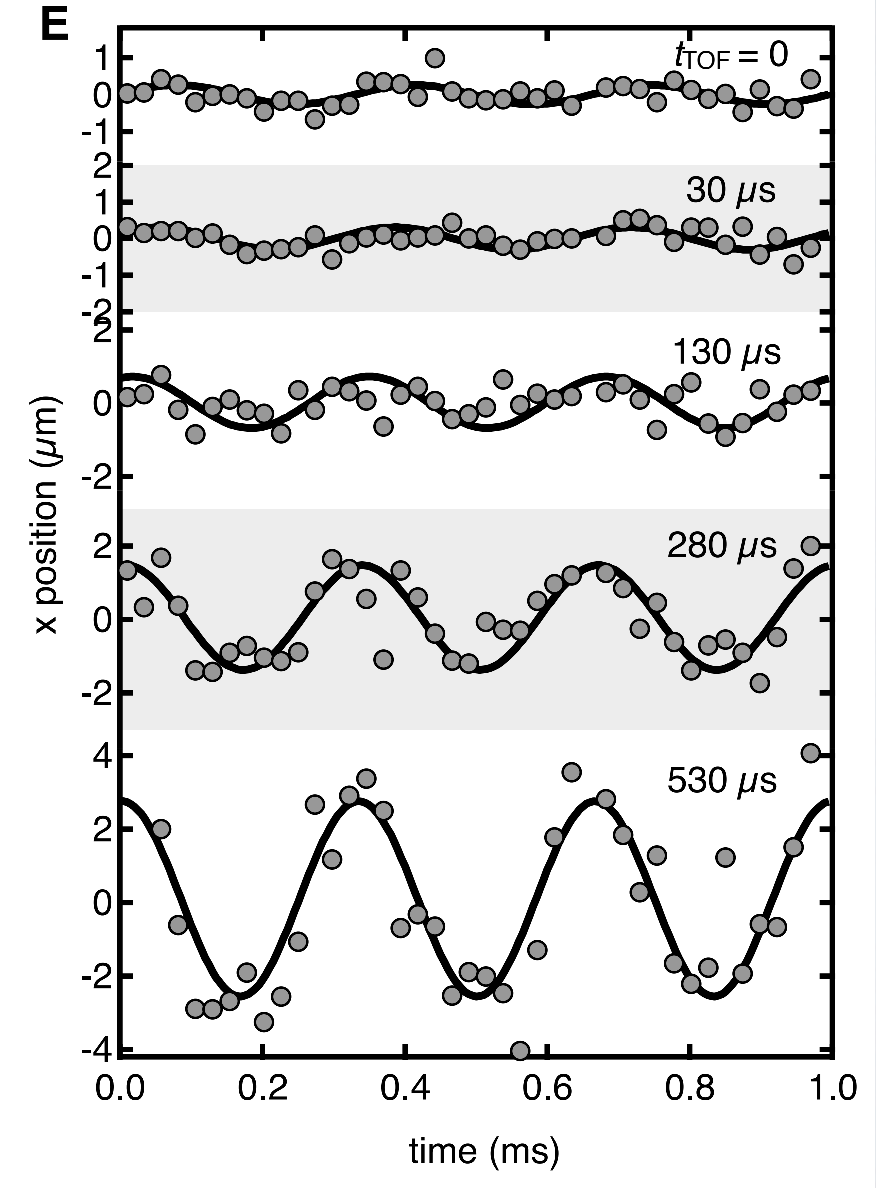




X

V

Ref.
L. J. Leblanc *et al* (in preparation)

ULTRASLOW RELATIVISTIC SYSTEM

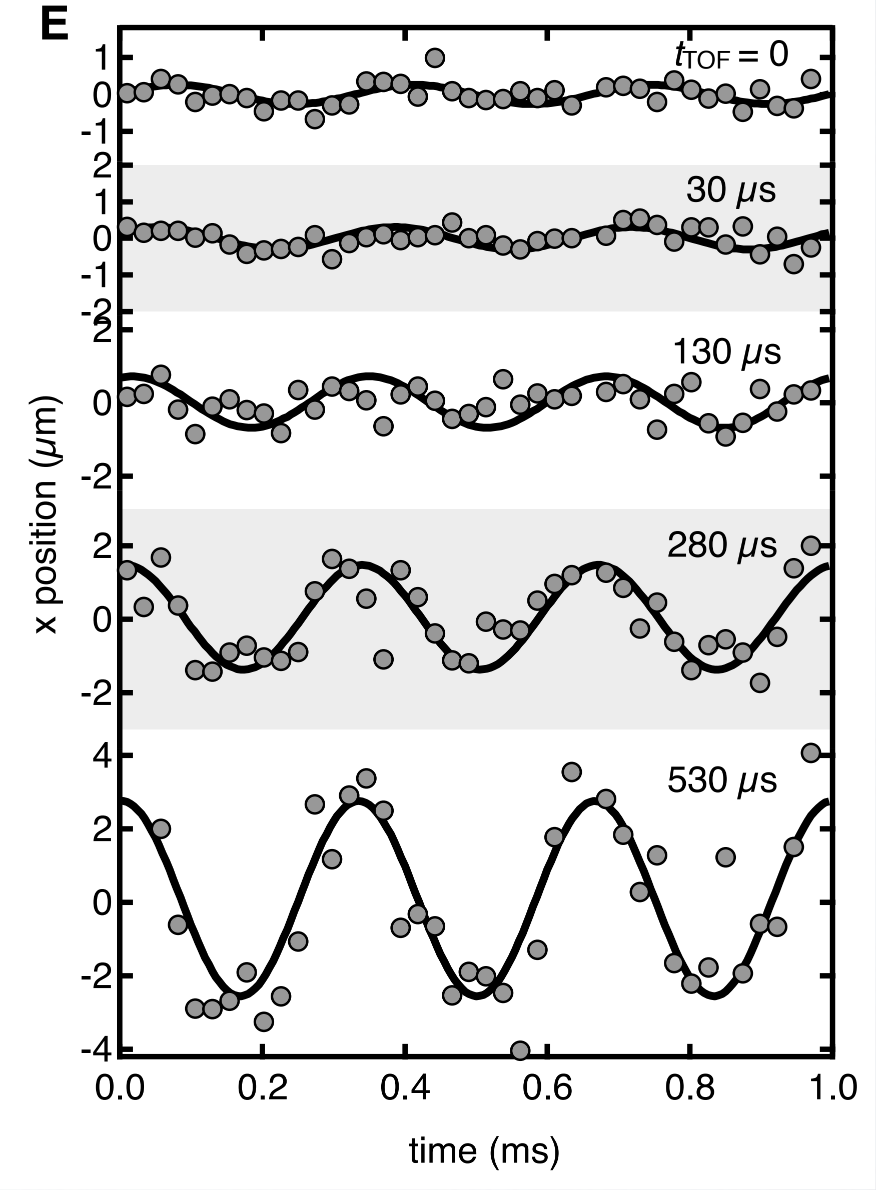
$$2mc^2 = \hbar\Omega_2 \approx h \times 1 \text{ kHz}$$

$$c = \hbar 2k_r / m_{\text{Rb}} \approx 11 \text{ mm/s}$$

$$\lambda_C = \frac{h}{mc}$$

$$\langle x(t) \rangle = \frac{\lambda_C}{4\pi} \sin\left(\frac{4\pi c}{\lambda_C} t\right)$$

$$\langle \dot{x}(t) \rangle = c \cos\left(\frac{4\pi c}{\lambda_C} t\right)$$



X

V

Ref.

L. J. Leblanc *et al* (in preparation)

ULTRASLOW RELATIVISTIC SYSTEM

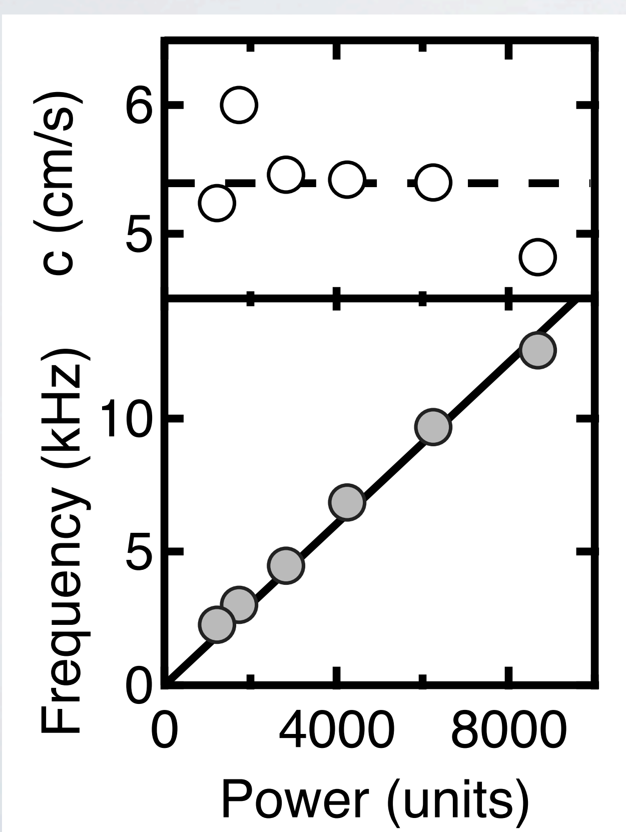
$$2mc^2 = \hbar\Omega_2 \approx h \times 1\ \mathrm{kHz}$$

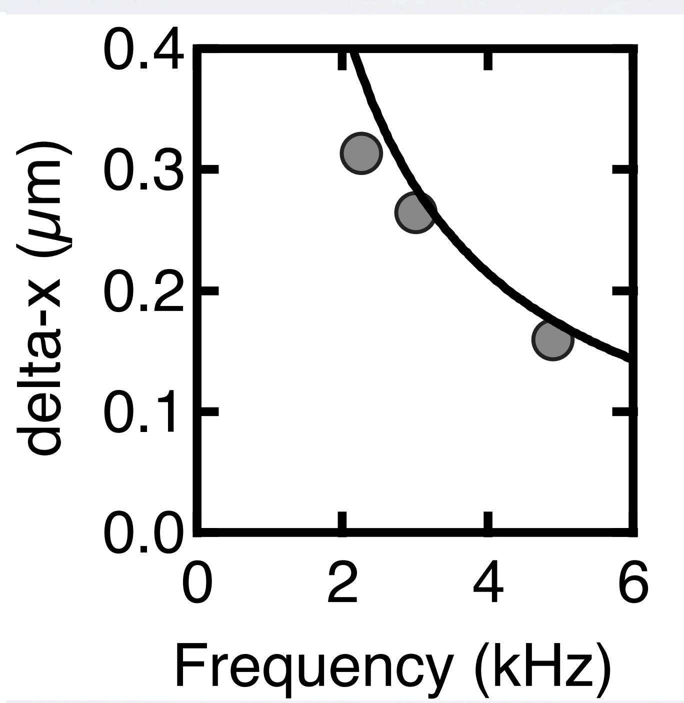
$$c = \hbar 2k_r / m_{\mathrm{Rb}} \approx 11\ \mathrm{mm/s}$$

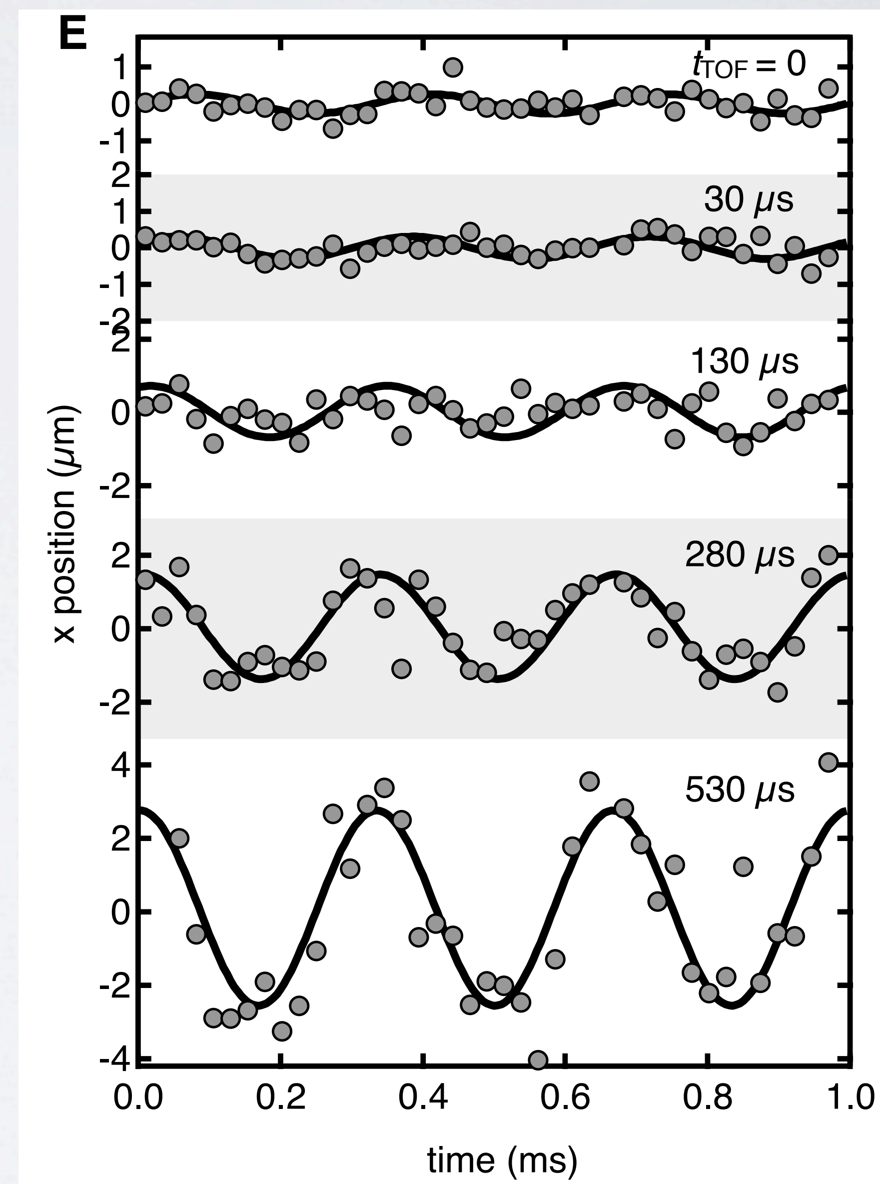
$$\lambda_C = \frac{h}{mc}$$

$$\langle x(t) \rangle = \frac{\lambda_C}{4\pi} \sin\left(\frac{4\pi c}{\lambda_C} t\right)$$

$$\langle \dot{x}(t) \rangle = c \cos\left(\frac{4\pi c}{\lambda_C} t\right)$$







X

V

Ref.

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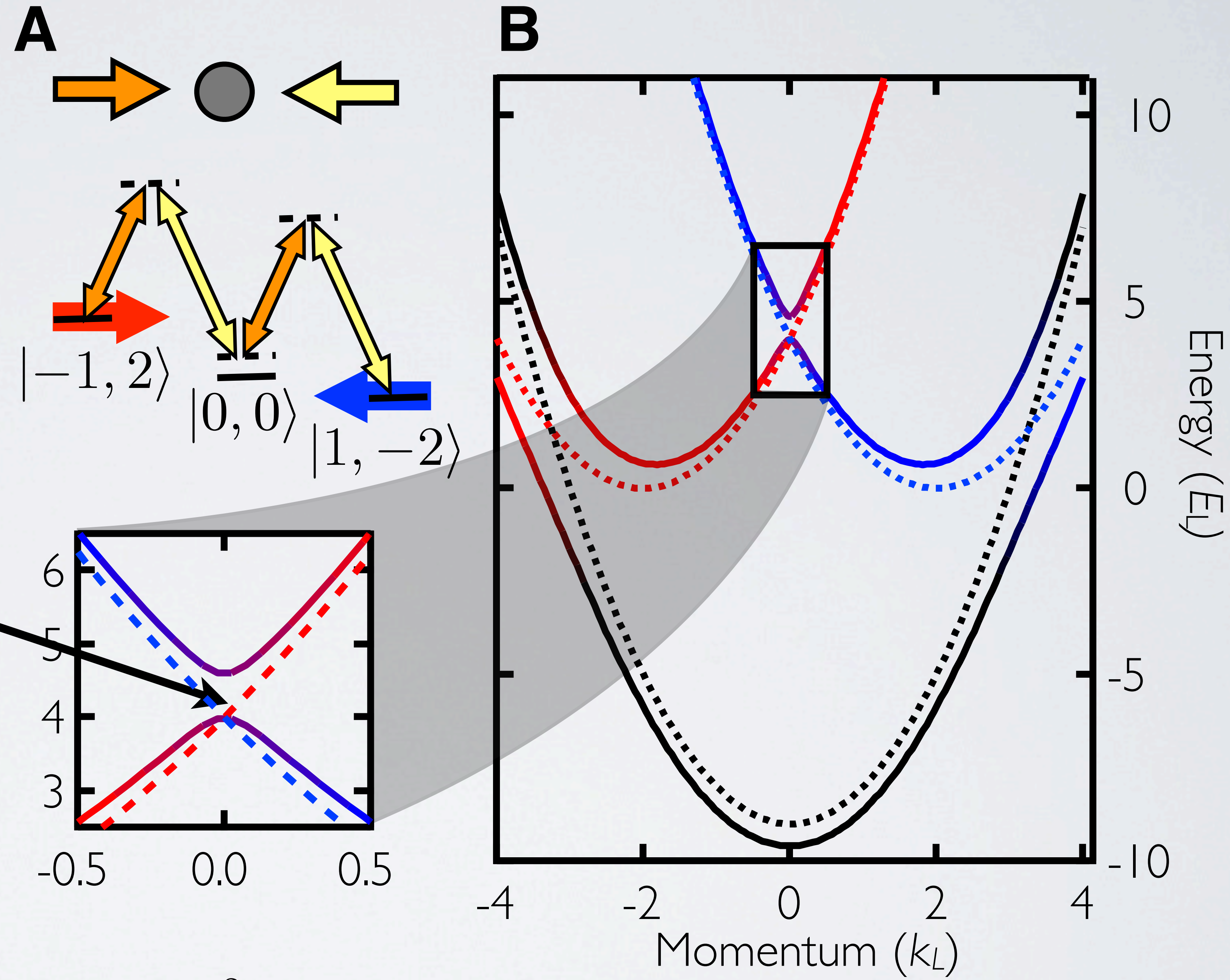
PHYSICAL INTERPRETATION



Start in pure "massless" state

$|\psi(t=0)\rangle = |\uparrow, k = 2k_r\rangle$

Evolve with coupling

$$|\psi(t)\rangle = \cos\left(\frac{mc^2}{\hbar}t\right)|\uparrow, k = 2k_r\rangle + i\sin\left(\frac{mc^2}{\hbar}t\right)|\downarrow, k = -2k_r\rangle$$

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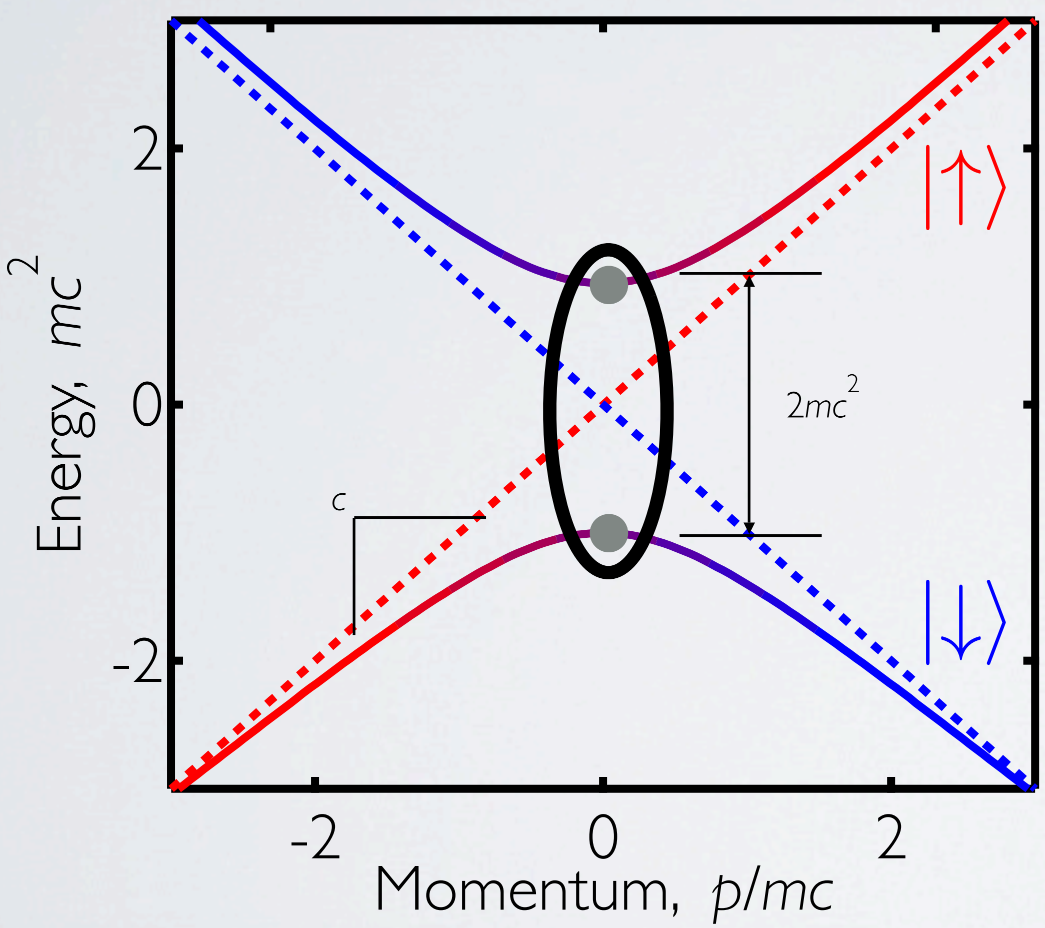
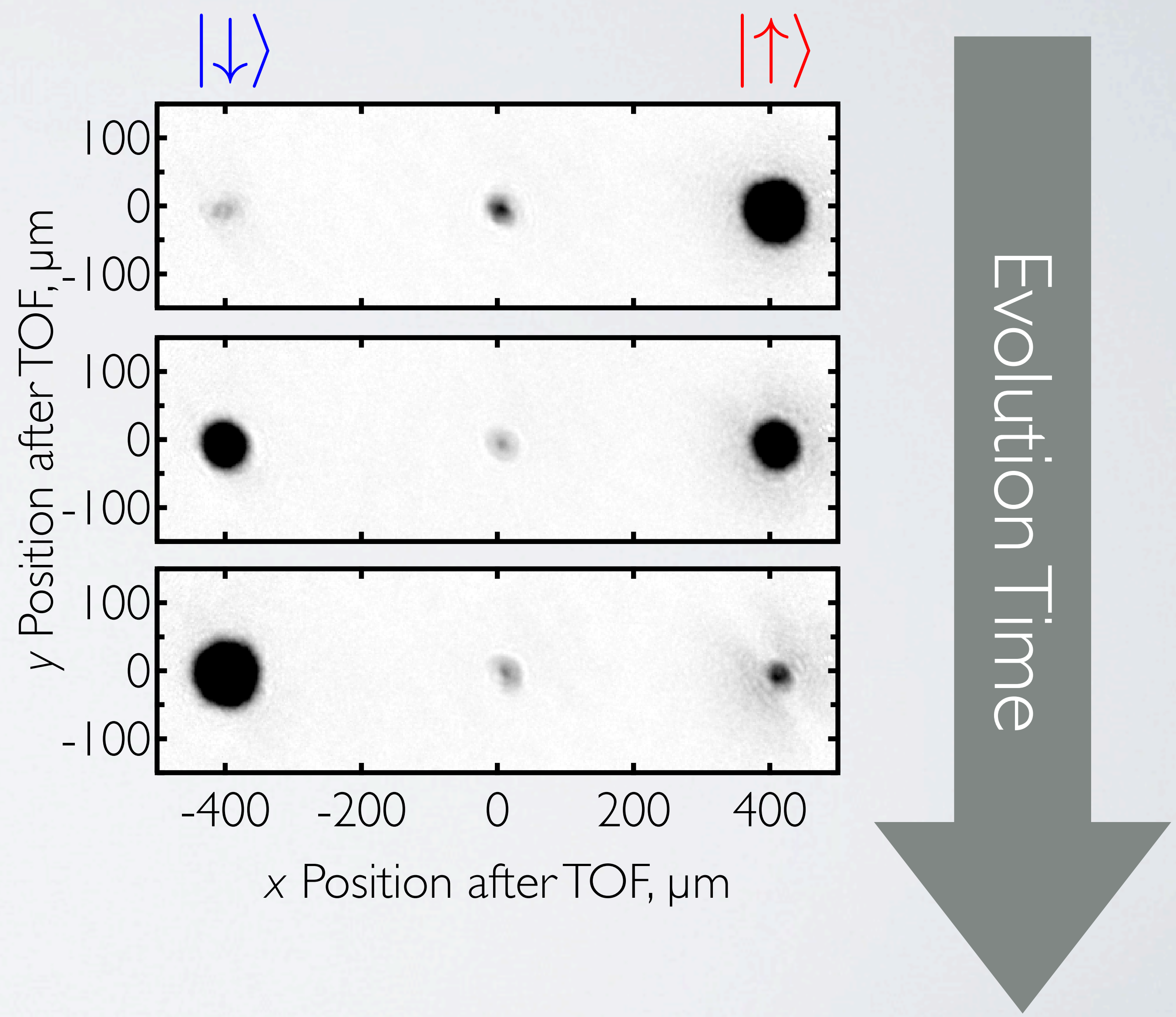
PHYSICAL INTERPRETATION

Rabi oscillations in a 2-level system

Two level system



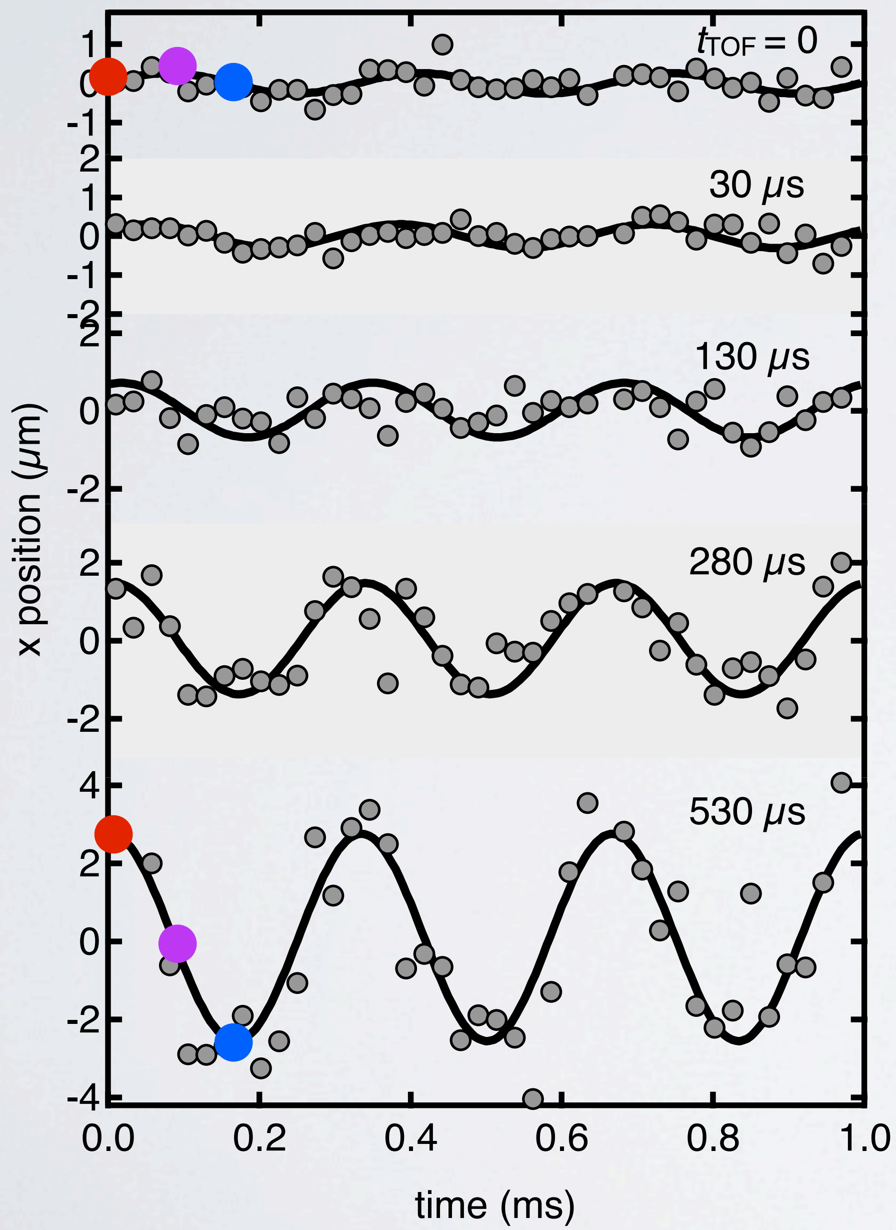
Time of flight images



PHYSICAL INTERPRETATION

Rabi oscillations in a 2-level system

Zitterbewegung



Time of flight images