Statistical Mechanics of Money, Income, Debt, and Energy Consumption

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with A. A. Dragulescu, A. C. Silva, A. Banerjee, T. Di Matteo, J. B. Rosser

- *European Physical Journal B* **17**, 723 (2000) ➔ …………… ➔
- *Reviews of Modern Physics* **81**, 1703 (2009)
- Book *Classical Econophysics* (Routledge, 2009)
- *New Journal of Physics* **12**, 075032 (2010).

Outline of the talk

- Statistical mechanics of money
- Debt and financial instability
- Two-class structure of income distribution
- Global inequality in energy consumption

"Money, it's a gas."

Pink Floyd

Boltzmann-Gibbs versus Pareto distribution

Ludwig Boltzmann (1844-1906)

Vilfredo Pareto (1848-1923)

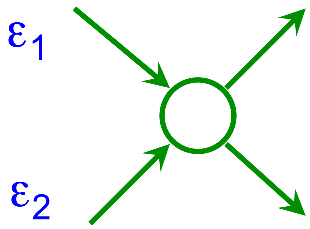
Boltzmann-Gibbs probability distribution $P(\varepsilon) \propto \exp(-\varepsilon/T)$, where ε is energy, and $T=\langle\varepsilon\rangle$ is temperature.

Pareto probability distribution $P(r) \propto 1/r^{(\alpha+1)}$ of income r.

An analogy between the distributions of energy ε and money m or income r

Boltzmann-Gibbs probability distribution of **money**

Collisions between atoms



$\varepsilon_1' = \varepsilon_1 + \Delta\varepsilon$

$\varepsilon_2' = \varepsilon_2 - \Delta\varepsilon$

Conservation of energy:

$\varepsilon_1 + \varepsilon_2 = \varepsilon_1' + \varepsilon_2'$

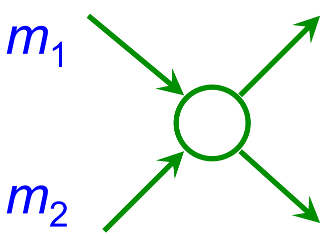
Detailed balance:

$\cancel{w_{12\to1'2'}} P(\varepsilon_1)\, P(\varepsilon_2) = \cancel{w_{1'2'\to12}} P(\varepsilon_1')\, P(\varepsilon_2')$

Boltzmann-Gibbs probability distribution $P(\varepsilon) \propto \exp(-\varepsilon/T)$ of energy ε, where $T = \langle\varepsilon\rangle$ is temperature. It is **universal** – independent of model rules, provided the model belongs to the time-reversal symmetry class.

Boltzmann-Gibbs distribution maximizes entropy $S = -\Sigma_\varepsilon P(\varepsilon) \ln P(\varepsilon)$ under the constraint of conservation law $\Sigma_\varepsilon P(\varepsilon)\, \varepsilon = \text{const.}$

Economic transactions between agents



$m_1' = m_1 + \Delta m$

$m_2' = m_2 - \Delta m$

Conservation of money:

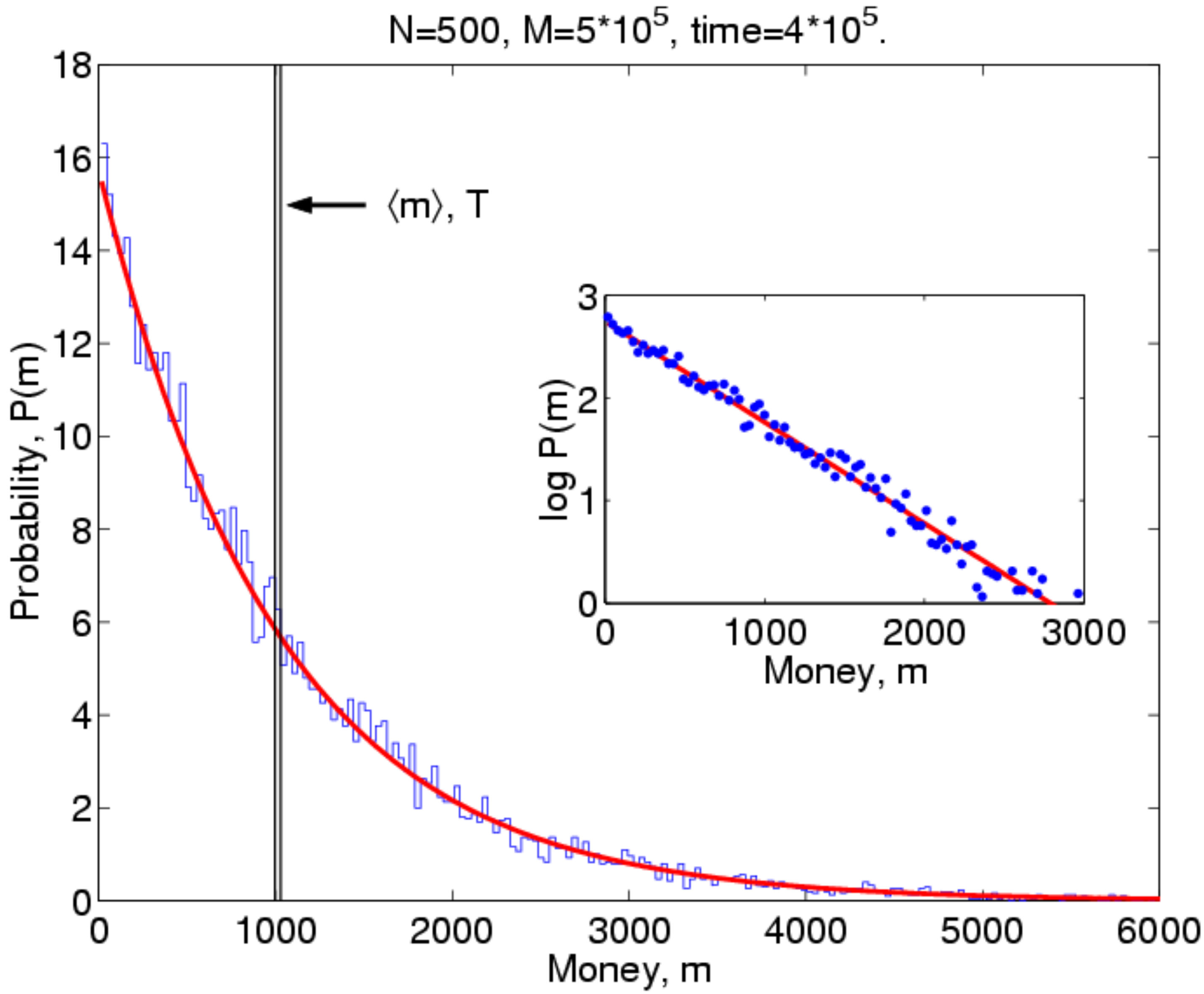
$m_1 + m_2 = m_1' + m_2'$

Detailed balance:

$w_{12\to1'2'} P(m_1)\, P(m_2) = w_{1'2'\to12} P(m_1')\, P(m_2')$

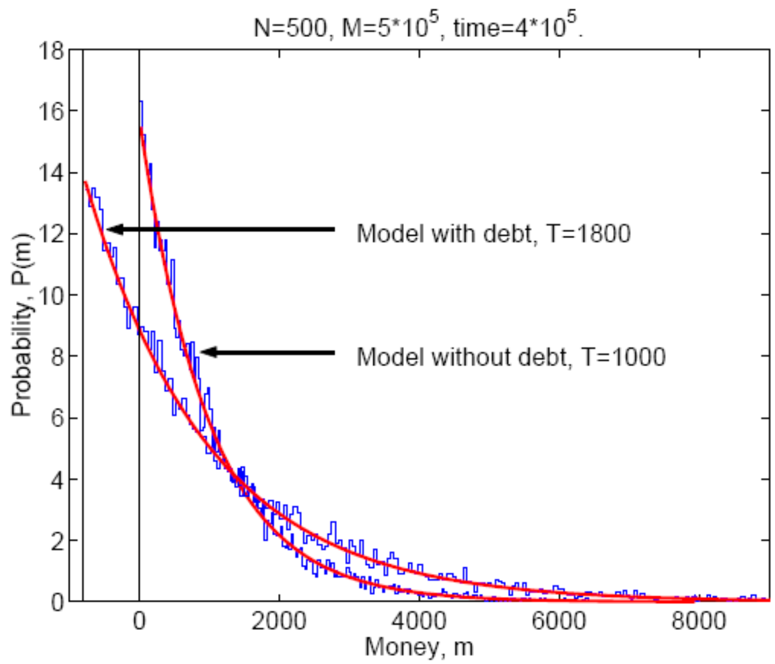
Boltzmann-Gibbs probability distribution $P(m) \propto \exp(-m/T)$ of money m, where $T = \langle m\rangle$ is the money temperature.

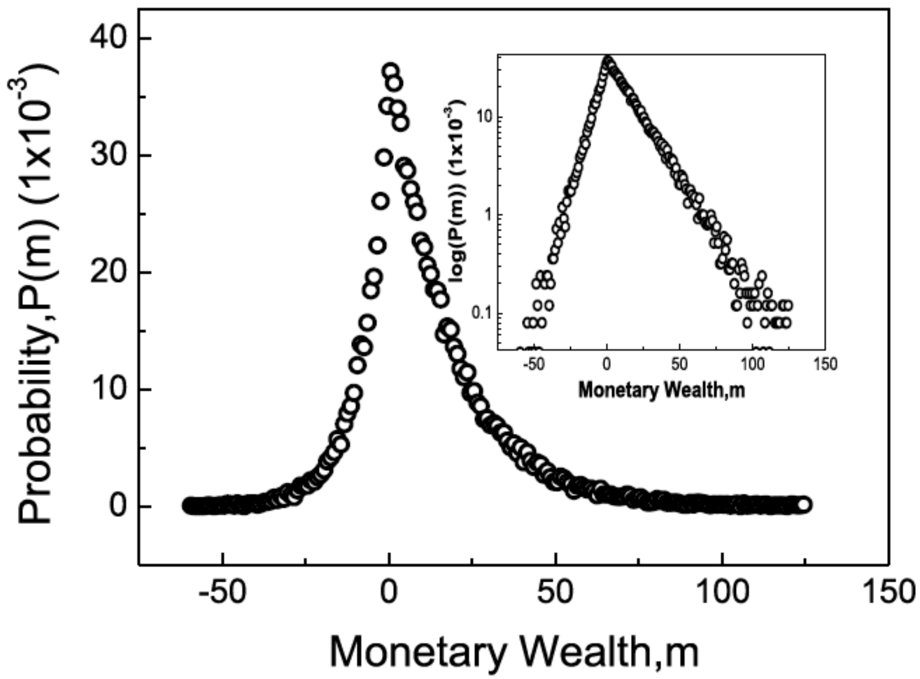
Computer simulation of money redistribution



The stationary distribution of money *m* is exponential: $P(m) \propto e^{-m/T}$

Money distribution with debt



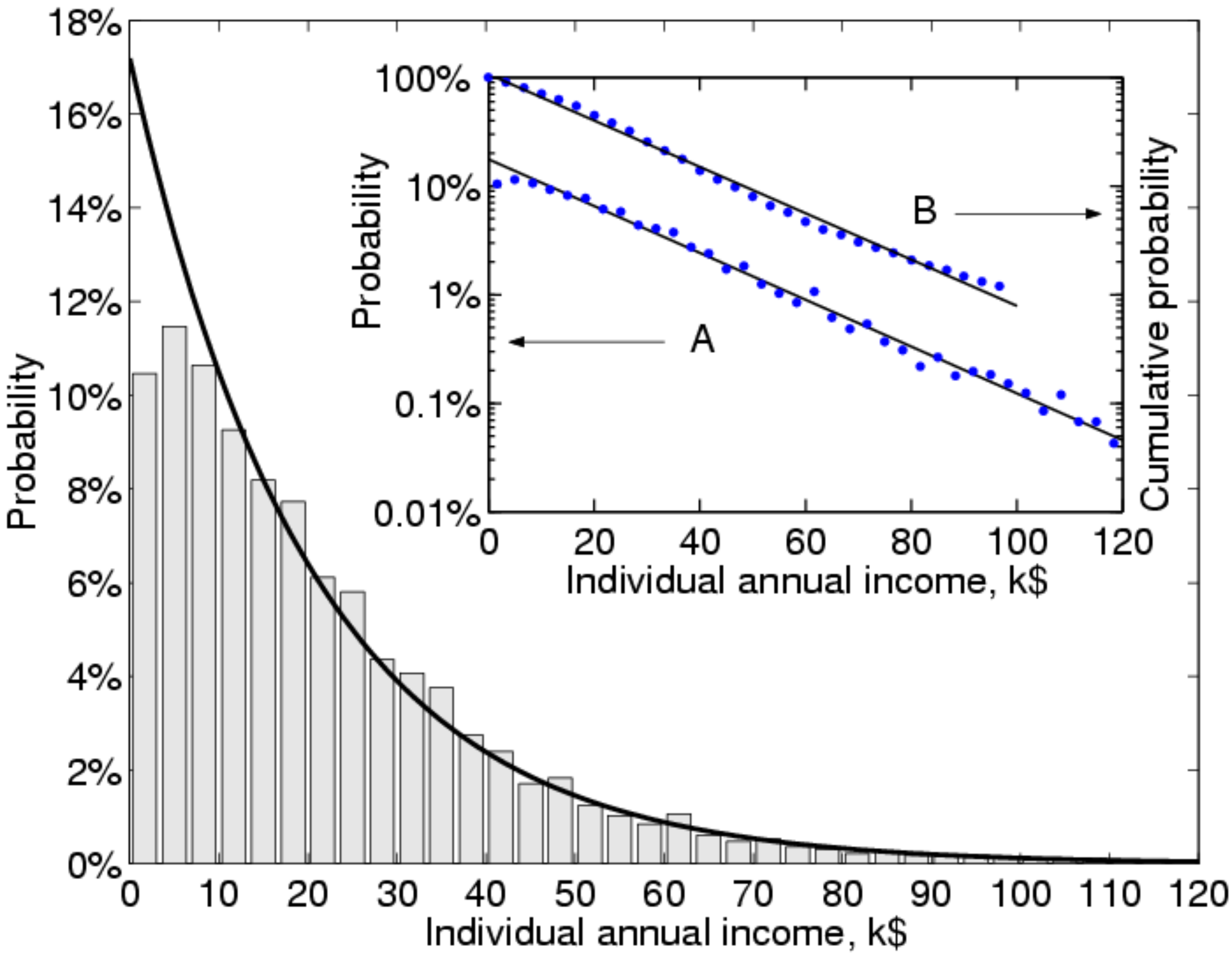


Debt per person is limited to 800 units.

Total debt in the system is limited via the Required Reserve Ratio (RRR):
Xi, Ding, Wang, *Physica A* **357**, 543 (2005)

- In practice, RRR is enforced inconsistently and does not limit total debt.
- Without a constraint on debt, the system does not have a stationary equilibrium.
- Free market itself does not have an intrinsic mechanism for limiting debt, and there is no such thing as the equilibrium debt.

Probability distribution of individual income

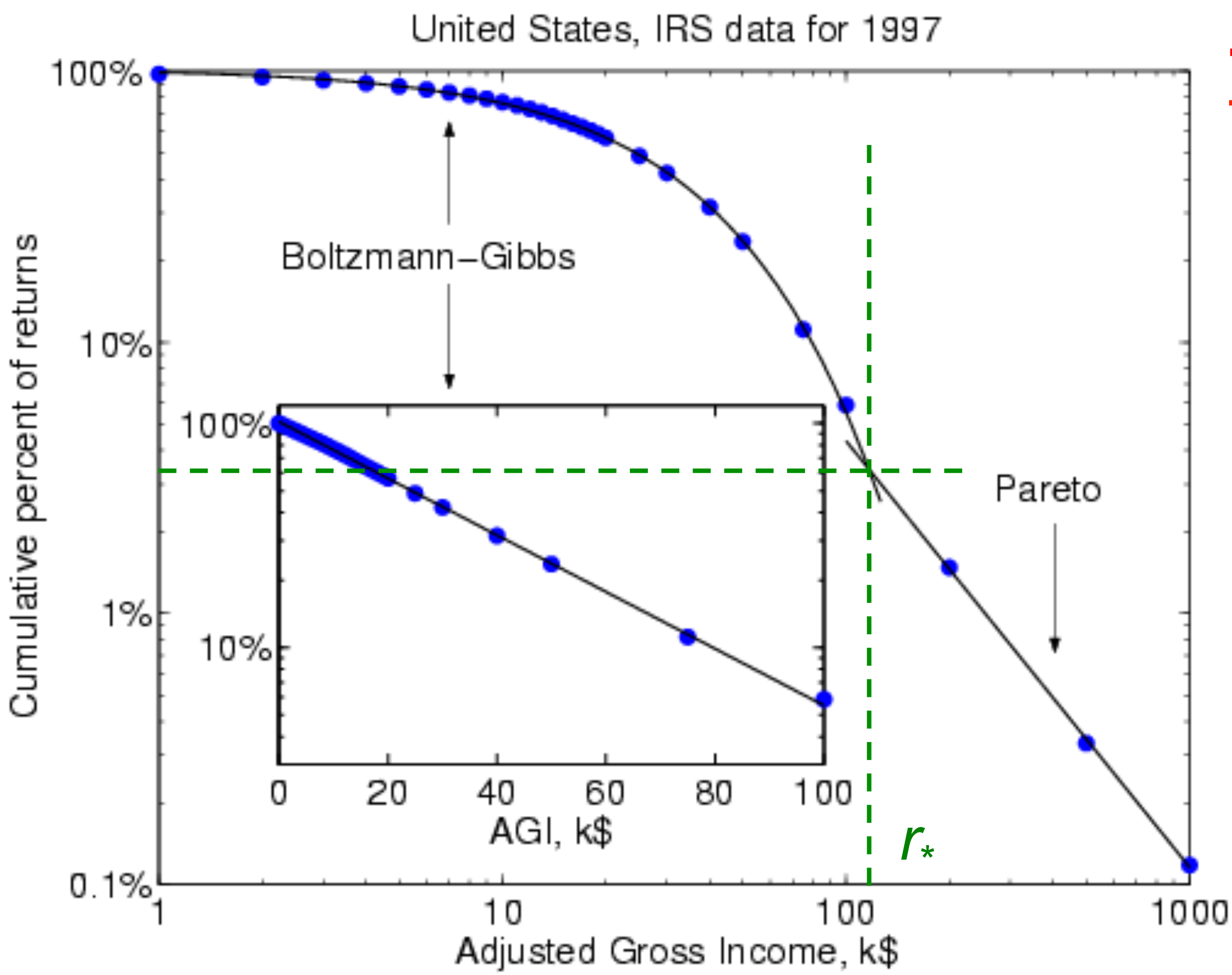


US Census data 1996 – histogram and points A

PSID: Panel Study of Income Dynamics, 1992 (U. Michigan) – points B

Distribution of income r is exponential: $P(r) \propto e^{-r/T}$

Income distribution in the USA, 1997



Two-class society

Upper Class

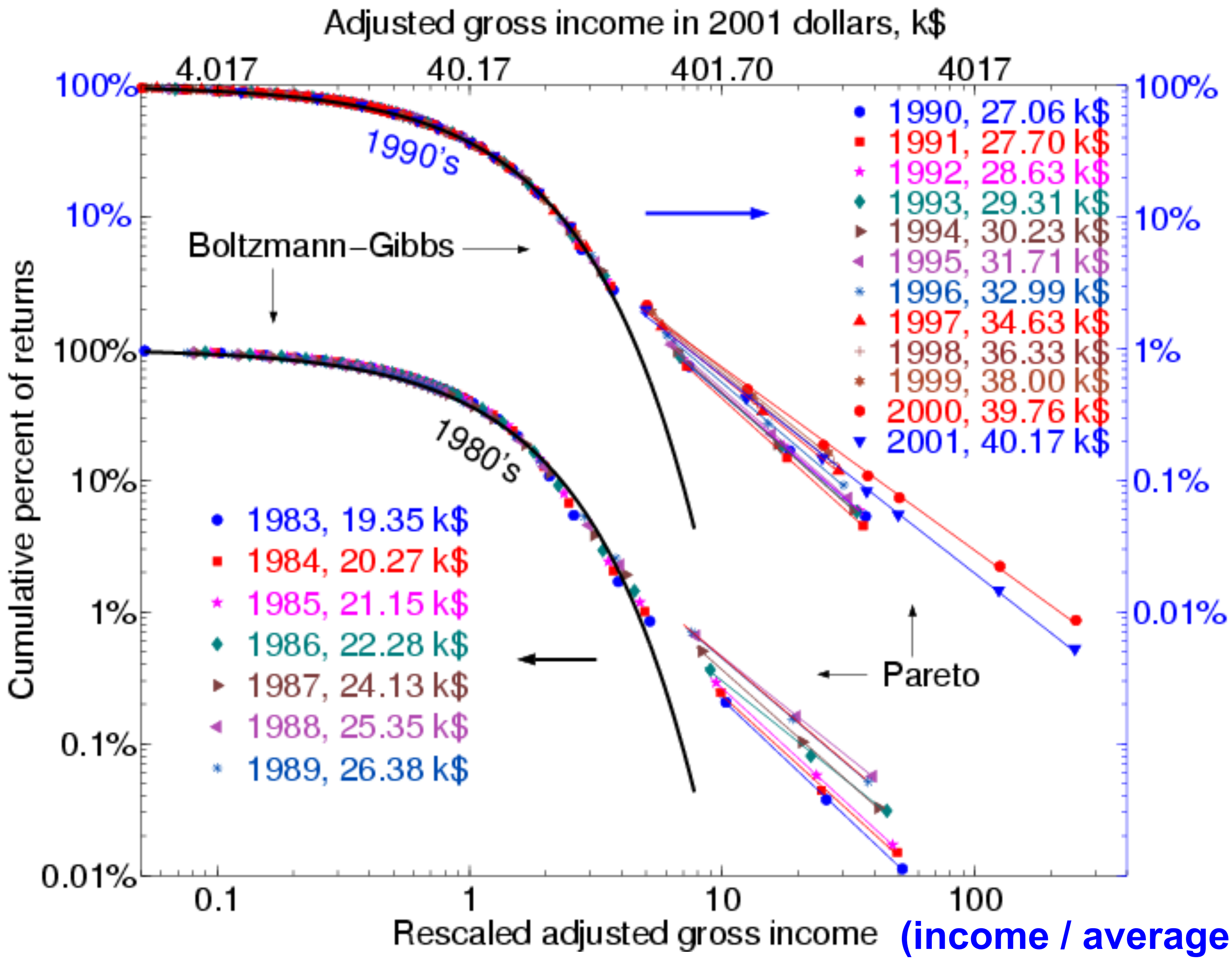
- Pareto power law
- 3% of population
- 16% of income
- Income > 120 k$: investments, capital

Lower Class

- Boltzmann-Gibbs exponential law
- 97% of population
- 84% of income
- Income < 120 k$: wages, salaries

"Thermal" bulk and "super-thermal" tail distribution

Income distribution in the USA, 1983-2001

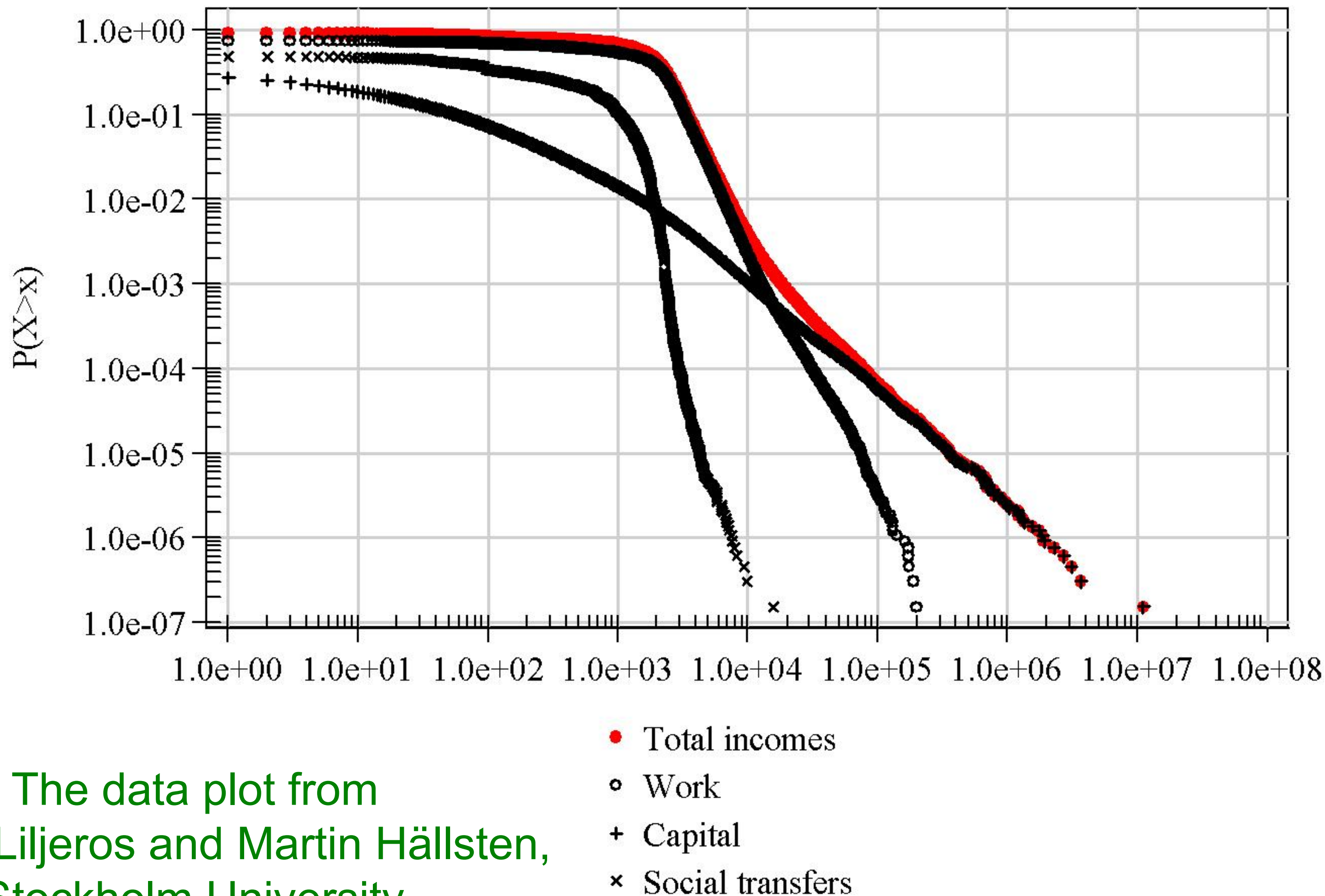


(income / average income *T*)

The rescaled exponential part does not change,

but the power-law part changes significantly.

Income distribution in Sweden



The data plot from
Fredrik Liljeros and Martin Hällsten,
Stockholm University

The origin of two classes

- Different sources of income: salaries and wages for the lower class, and capital gains and investments for the upper class.
- Their income dynamics can be described by additive and multiplicative diffusion, correspondingly.
- From the social point of view, these can be the classes of employees and employers, as described by Karl Marx.
- Emergence of classes from the initially equal agents was simulated by Ian Wright "The Social Architecture of Capitalism" *Physica A* **346**, 589 (2005), see also the new book "Classical Econophysics" (2009)

Diffusion model for income kinetics

Suppose income changes by small amounts Δr over time Δt.
Then $P(r,t)$ satisfies the Fokker-Planck equation for $0<r<\infty$:

$$\frac{\partial P}{\partial t} = \frac{\partial}{\partial r}\left(AP + \frac{\partial}{\partial r}(BP)\right), \quad A = -\left\langle \frac{\Delta r}{\Delta t} \right\rangle, \quad B = \left\langle \frac{(\Delta r)^2}{2\Delta t} \right\rangle.$$

For a stationary distribution, $\partial_t P = 0$ and $\frac{\partial}{\partial r}(BP) = -AP$.

For the lower class, Δr are independent of r – additive diffusion, so A and B are constants. Then, $P(r) \propto \exp(-r/T)$, where $T = B/A$, – an exponential distribution.

For the upper class, $\Delta r \propto r$ – multiplicative diffusion, so $A = ar$ and $B = br^2$. Then, $P(r) \propto 1/r^{\alpha+1}$, where $\alpha = 1+a/b$, – a power-law distribution.

For the upper class, income does change in percentages, as shown by Fujiwara, Souma, Aoyama, Kaizoji, and Aoki (2003) for the tax data in Japan. For the lower class, the data is not known yet.

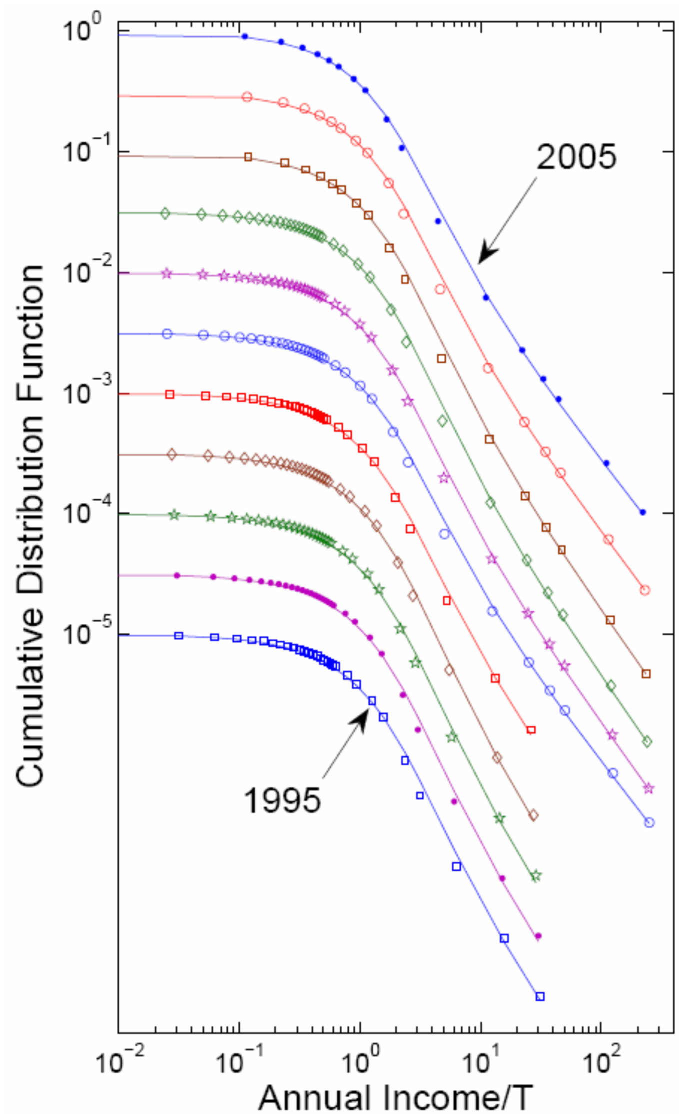
Additive and multiplicative income diffusion

If the additive and multiplicative diffusion processes are present simultaneously, then $A = A_0 + ar$ and $B = B_0 + br^2 = b(r_0^2 + r^2)$. The stationary solution of the FP equation is

$$P(r) = \frac{C\, e^{-\frac{r_0}{T}\arctan\left(\frac{r}{r_0}\right)}}{\left[1+(r/r_0)^2\right]^{1+a/2b}}$$

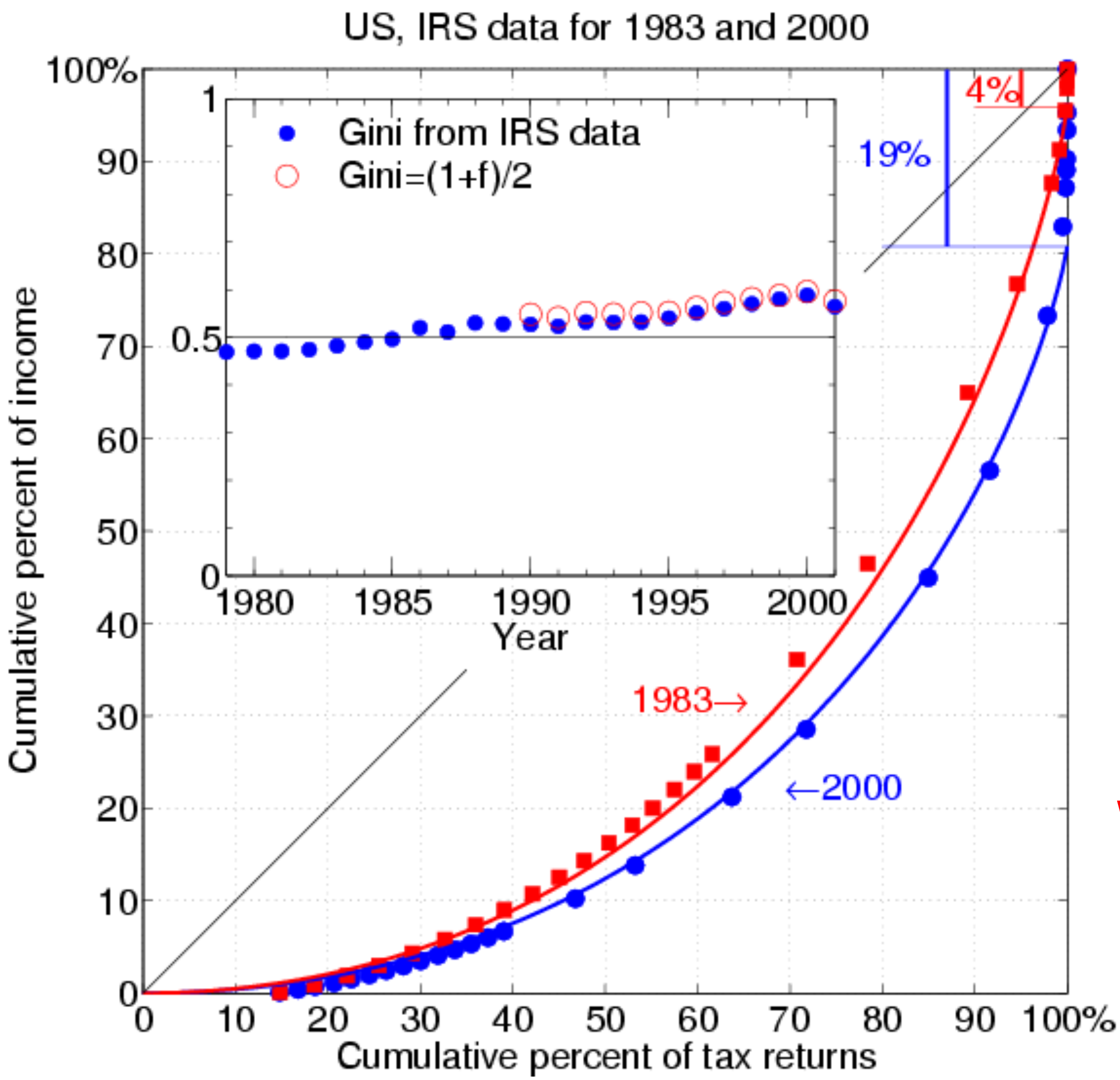
It interpolates between the exponential and the power-law distributions and has 3 parameters:

- $T = B_0/A_0$ – the temperature of the exponential part
- $\alpha = 1 + a/b$ – the power-law exponent of the upper tail
- r_0 – the crossover income between the lower and upper parts.



Yakovenko (2007) arXiv:0709.3662, Fiaschi and Marsili (2007) preprint online

Lorenz curves and income inequality



Lorenz curve ($0<r<\infty$):

$$x(r)=\int_0^r P(r')\,dr'$$

$$y(r)=\int_0^r r'P(r')\,dr'\Big/\langle r'\rangle$$

A measure of inequality, the Gini coefficient is G =

$$\frac{\text{Area(diagonal line - Lorenz curve)}}{\text{Area(Triangle beneath diagonal)}}$$

For exponential distribution, G=1/2 and the Lorenz curve is

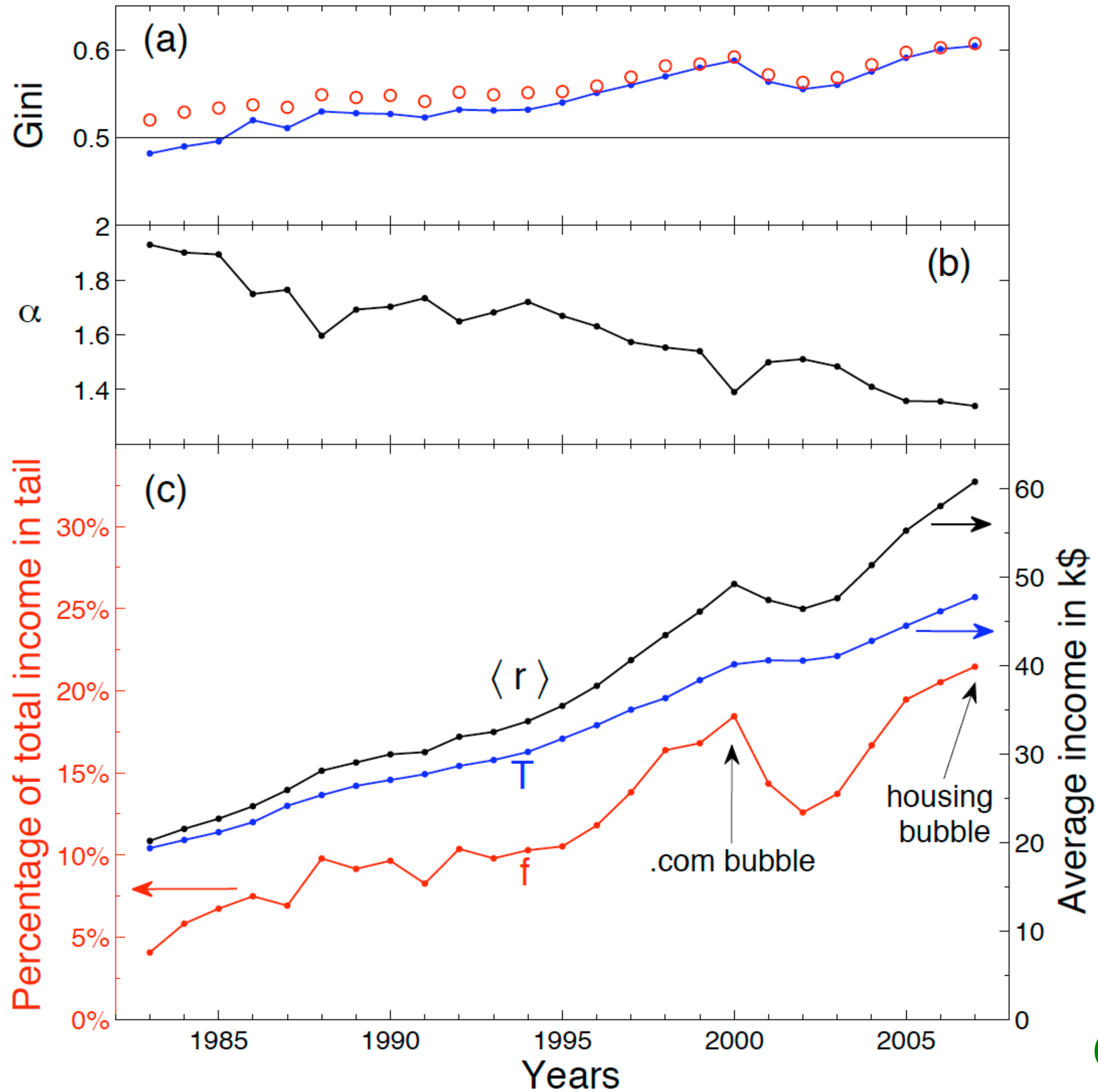
$$y = x + (1-x)\ln(1-x)$$

With a tail, the Lorenz curve is

$$y=(1-f)[x+(1-x)\ln(1-x)] + f\,\Theta(x-1),$$

where f is the tail income, and Gini coefficient is $G=(1+f)/2$.

Time evolution of income inequality



Income inequality peaks during speculative bubbles in financial markets

$$f = \frac{\langle r \rangle - T}{\langle r \rangle}$$

f - fraction of income in the tail

<r> − average income in the whole system

T − average income in the exponential part

Gini coefficient G=(1+*f*)/2

“The next great depression will be from 2008 to 2023”

Harry S. Dent, book “The Great Boom Ahead”, page 16, published in 1993

His forecast was based on demographic data: The post-war ”baby boomers” generation to invest retirement savings in the stock market massively in the 1990s.

His new book “The Great Depression Ahead”, January 2009

The current financial crisis is not the only and, perhaps, not the most important crises that the mankind faces:

- exhaustion of fossil fuels and other natural resources
- global warming caused by CO_2 emissions from fossil fuels

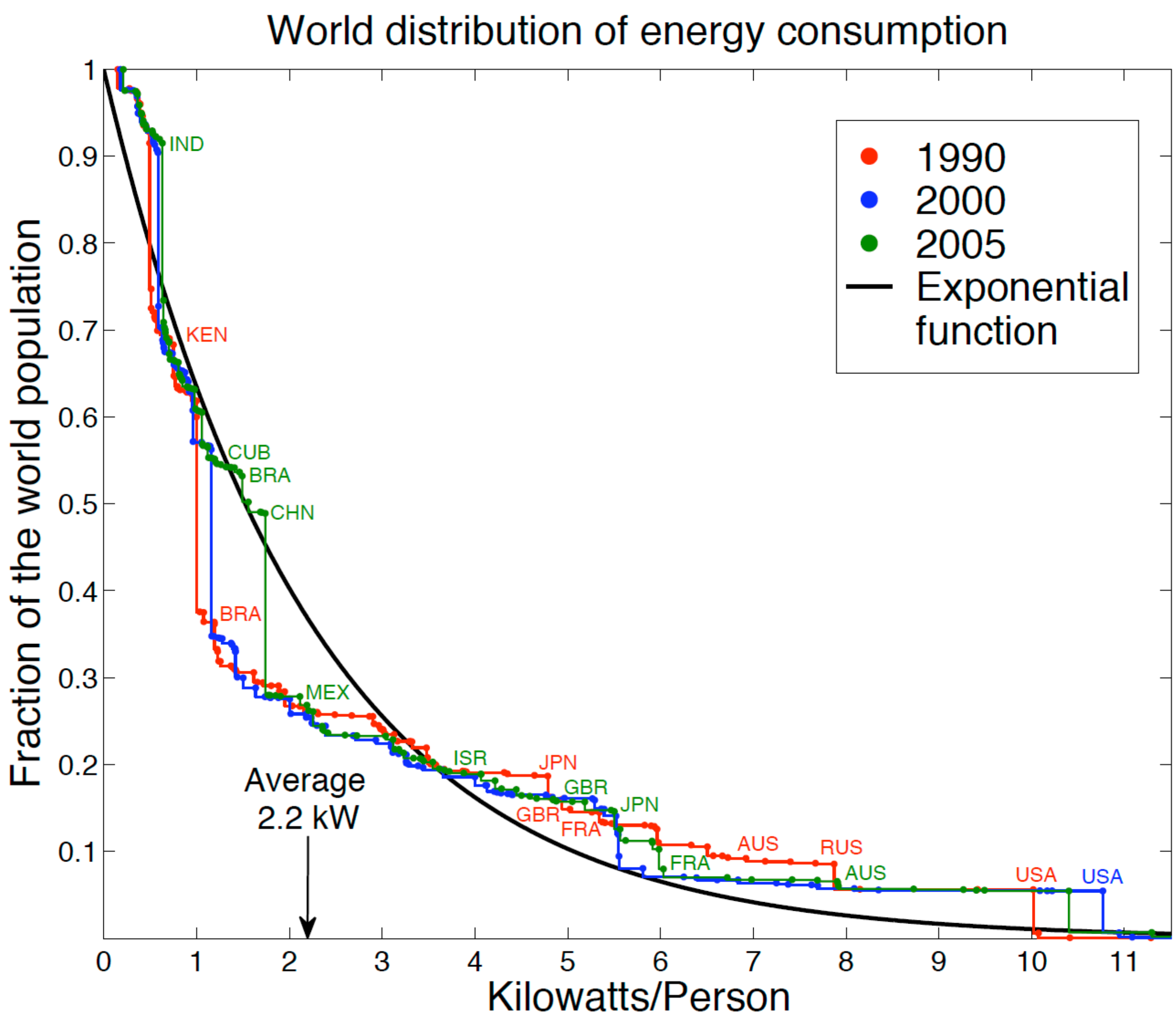
Brief history of the biosphere evolution:

- Plants consume and store energy from the Sun through photosynthesis
- Animals eat plants, which store Sun's energy
- Animals eat animals, which eat plants, which store Sun's energy
- Humans eat all of the above,

  + consume dead plants and animals (fossil fuels), which store Sun's energy

- For thousands of years, the progress of human civilization was biologically limited by muscle energy (of humans or animals) and by wood fuel.
- Couple of centuries ago, the humans discovered how to massively utilize Sun's energy stored in fossil fuels (coal and oil): the era of industrial revolution and modern capitalism.
- In a couple of centuries, the humans managed to spend fossil fuels accumulated for millions of years.
- Now this energy binge is coming to an end. Will humankind manage to find a new way for sustainable life? Will new technology save us?

Global inequality in energy consumption

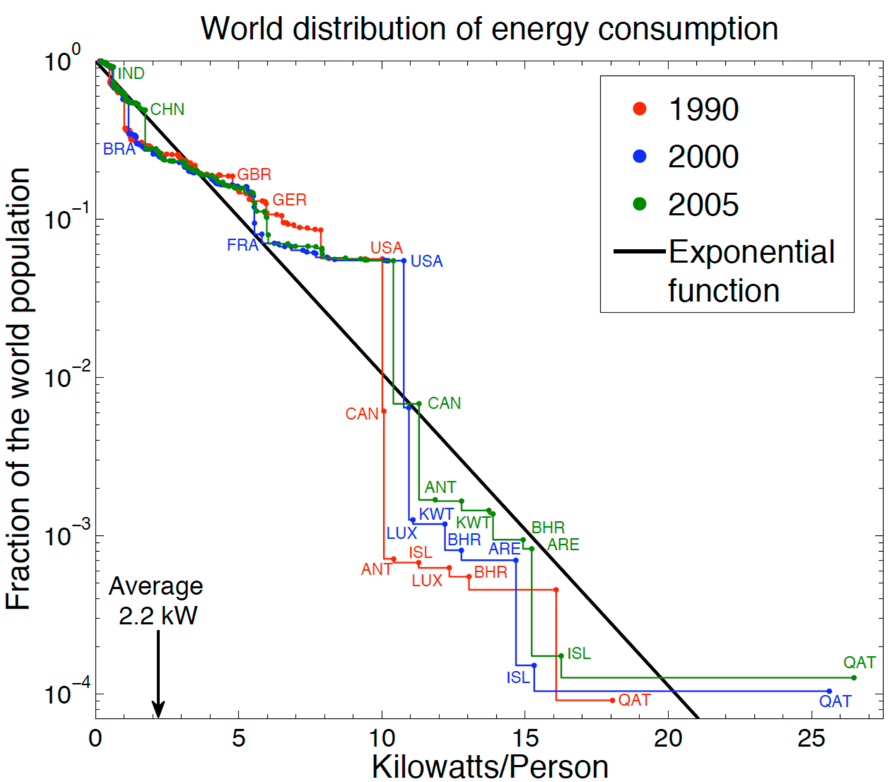


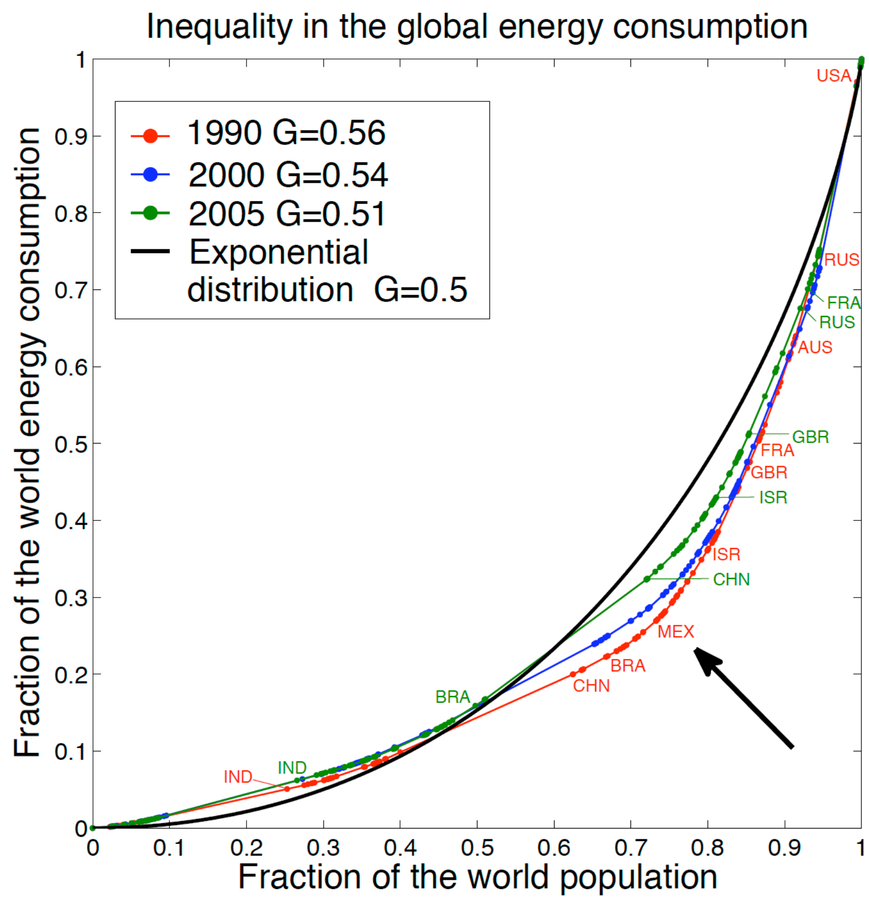
Global distribution of energy consumption per person is roughly exponential.

Division of a limited resource + entropy maximization produce exponential distribution.

Physiological energy consumption of a human at rest is about 100 W

Global inequality in energy consumption





The distribution is getting smoother with time. The gap in energy consumption between developed and developing countries shrinks.

The global inequality of energy consumption decreased from 1990 to 2005.

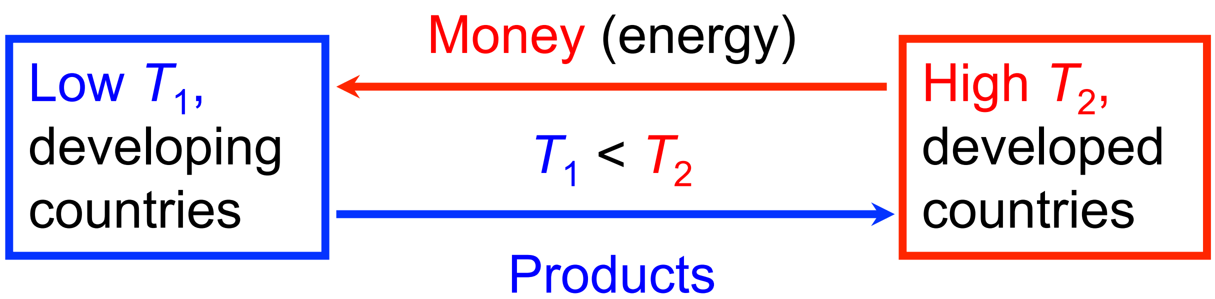
The energy consumption distribution is getting closer to the exponential.

Conclusions

- The probability distribution of money is stable and has an equilibrium only when a boundary condition, such as $m>0$, is imposed.
- When debt is permitted, the distribution of money becomes unstable, unless some sort of a limit on maximal debt is imposed.
- Income distribution in the USA has a two-class structure: exponential ("thermal") for the great majority (97-99%) of population and power-law ("superthermal") for the top 1-3% of population.
- The exponential part of the distribution is very stable and does not change in time, except for a slow increase of temperature T (the average income).
- The power-law tail is not universal and was increasing significantly for the last 20 years. It peaked and crashed in 2000 and 2006 with the speculative bubbles in financial markets.
- The global distribution of energy consumption per person is highly unequal and roughly exponential. This inequality is important in dealing with the global energy problems.

Thermal machine in the world economy

In general, different countries have different temperatures T, which makes possible to construct a thermal machine:



Prices are commensurate with the income temperature T (the average income) in a country.

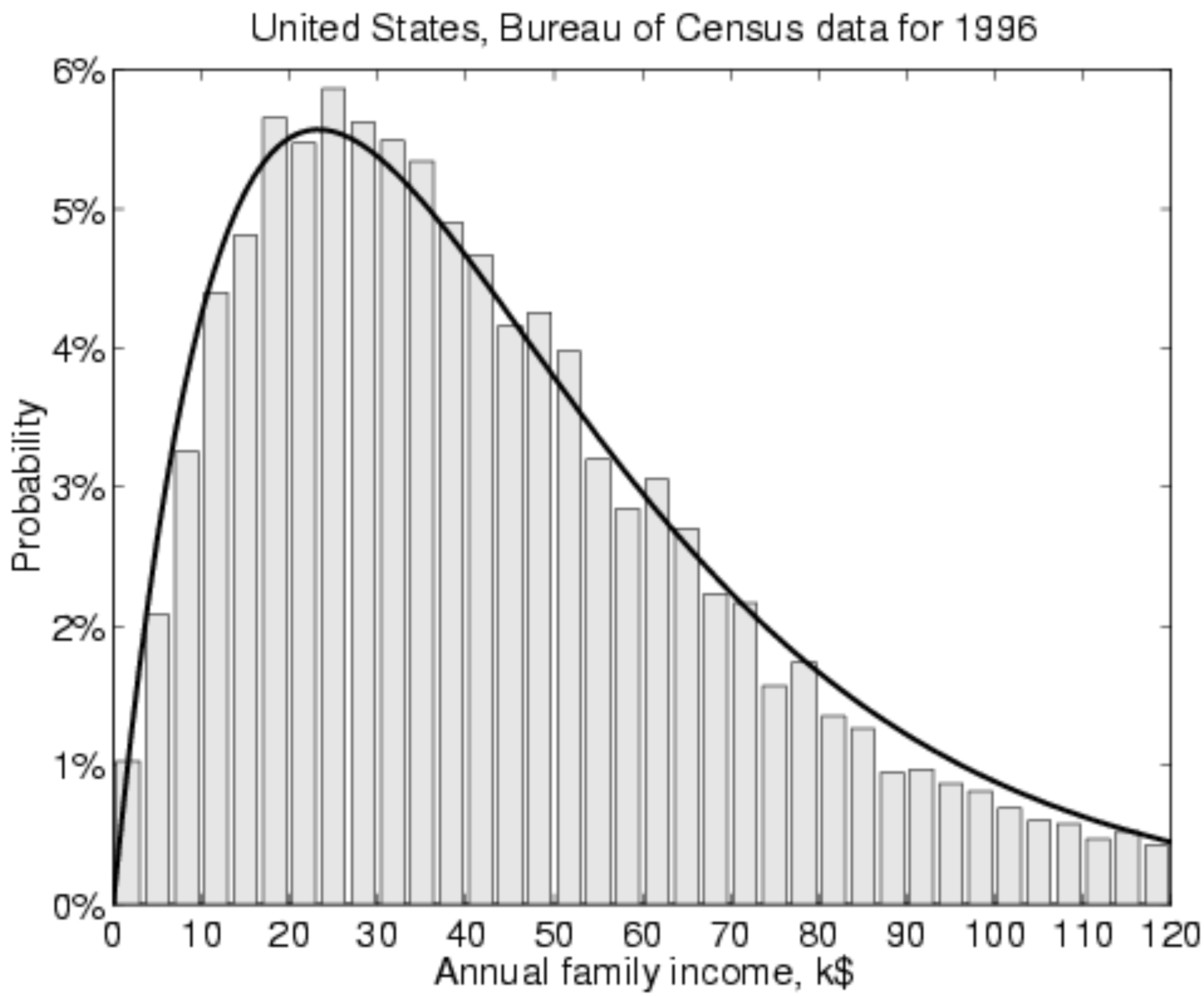
Products can be manufactured in a low-temperature country at a low price T_1 and sold to a high-temperature country at a high price T_2.

The **temperature difference** $T_2 – T_1$ is the **profit** of an intermediary.

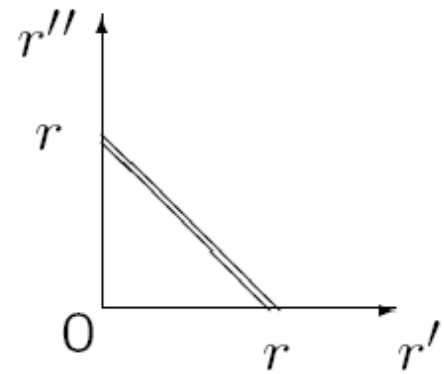
Money (energy) flows from high T_2 to low T_1 (the 2nd law of thermodynamics – entropy always increases) ⇔ **Trade deficit**

In full equilibrium, $T_2 = T_1$ ⇔ No profit ⇔ "Thermal death" of economy

Income distribution for two-earner families

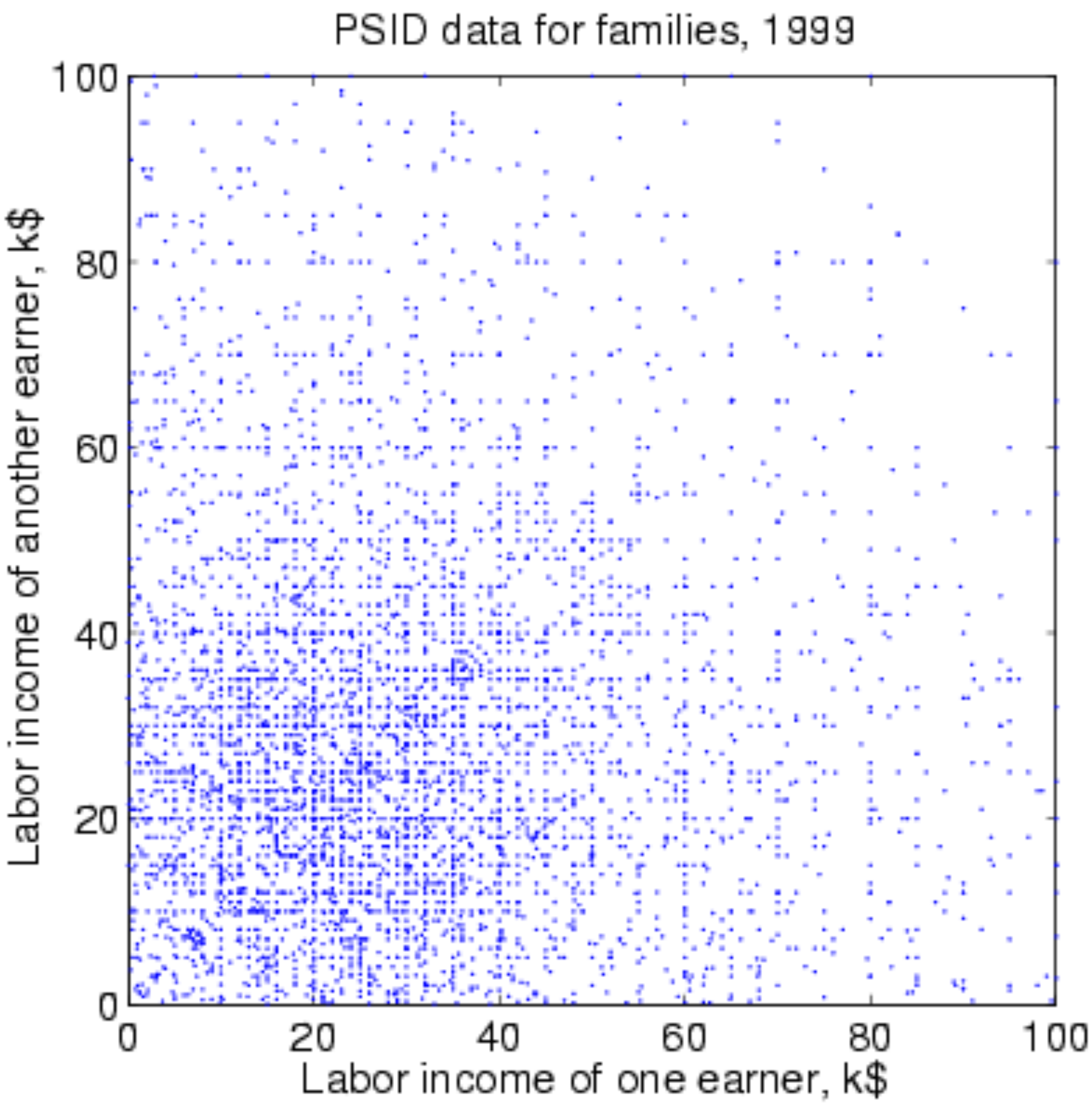


$$r = r' + r'', \quad P_2(r) = \int_0^r P_1(r') P_1(r - r') dr' \propto r \exp(-r/T)$$



The average family income is $2T$. The most probable family income is T.

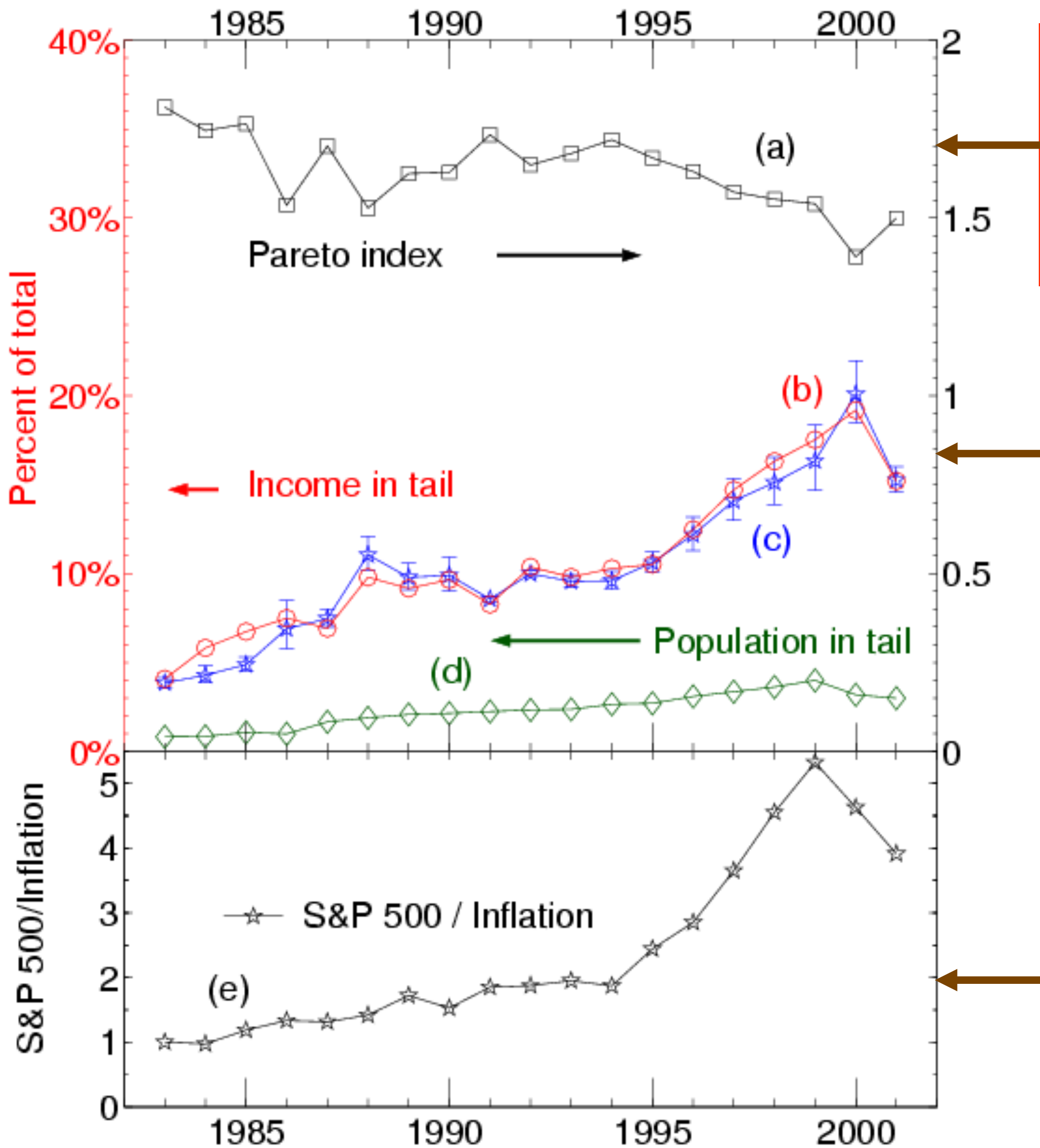
No correlation in the incomes of spouses



Every family is represented by two points (r_1, r_2) and (r_2, r_1).

The absence of significant clustering of points (along the diagonal) indicates that the incomes r_1 and r_2 are approximately uncorrelated.

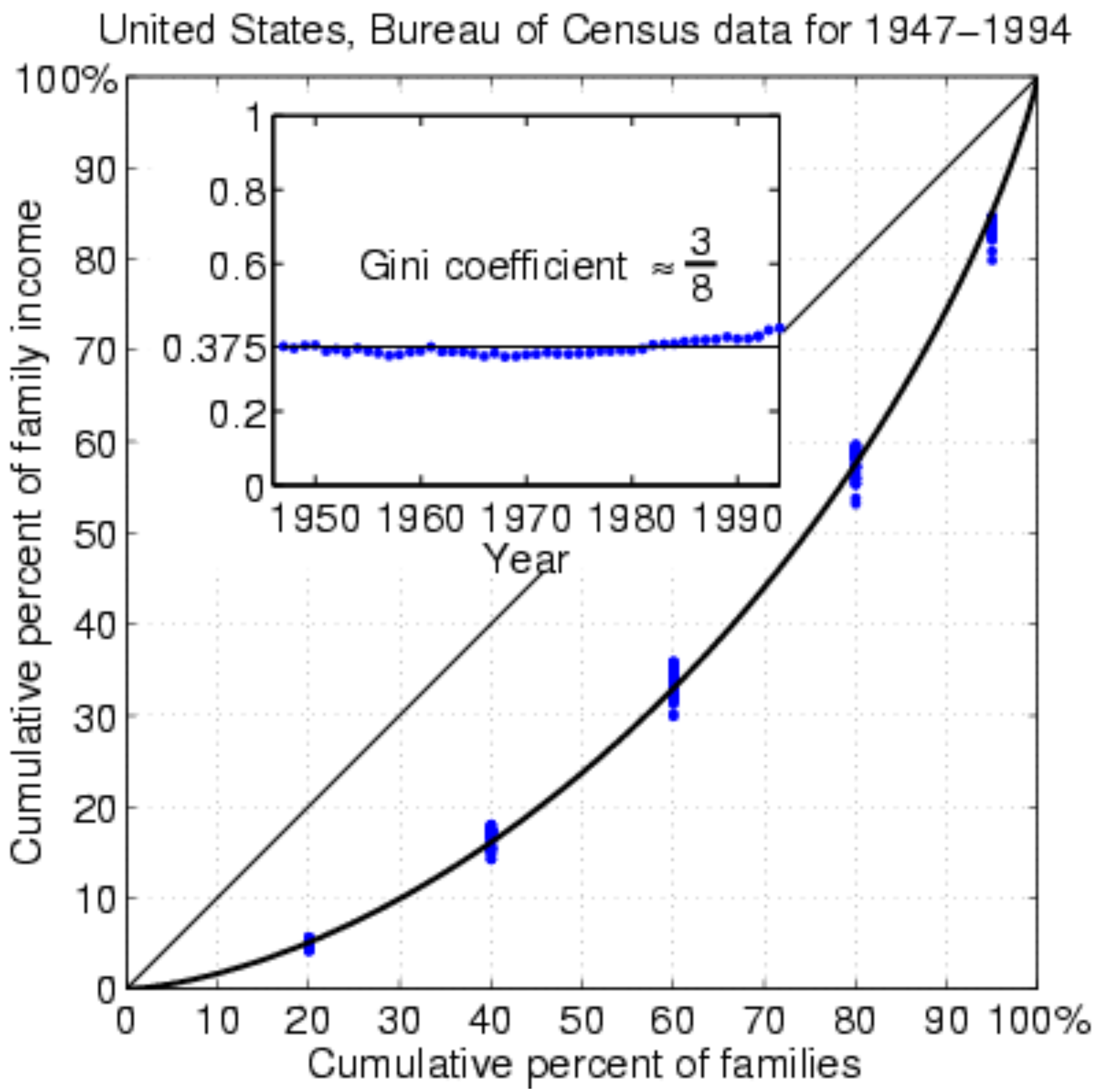
Time evolution of the tail parameters



The Pareto index α in $C(r) \propto 1/r^{\alpha}$ is non-universal.
It changed from 1.7 in 1983 to 1.3 in 2000.

- Pareto tail changes in time non-monotonously, in line with the stock market.
- The tail income swelled 5-fold from 4% in 1983 to 20% in 2000.
- It decreased in 2001 with the crash of the U.S. stock market.

Lorenz curve and Gini coefficient for families

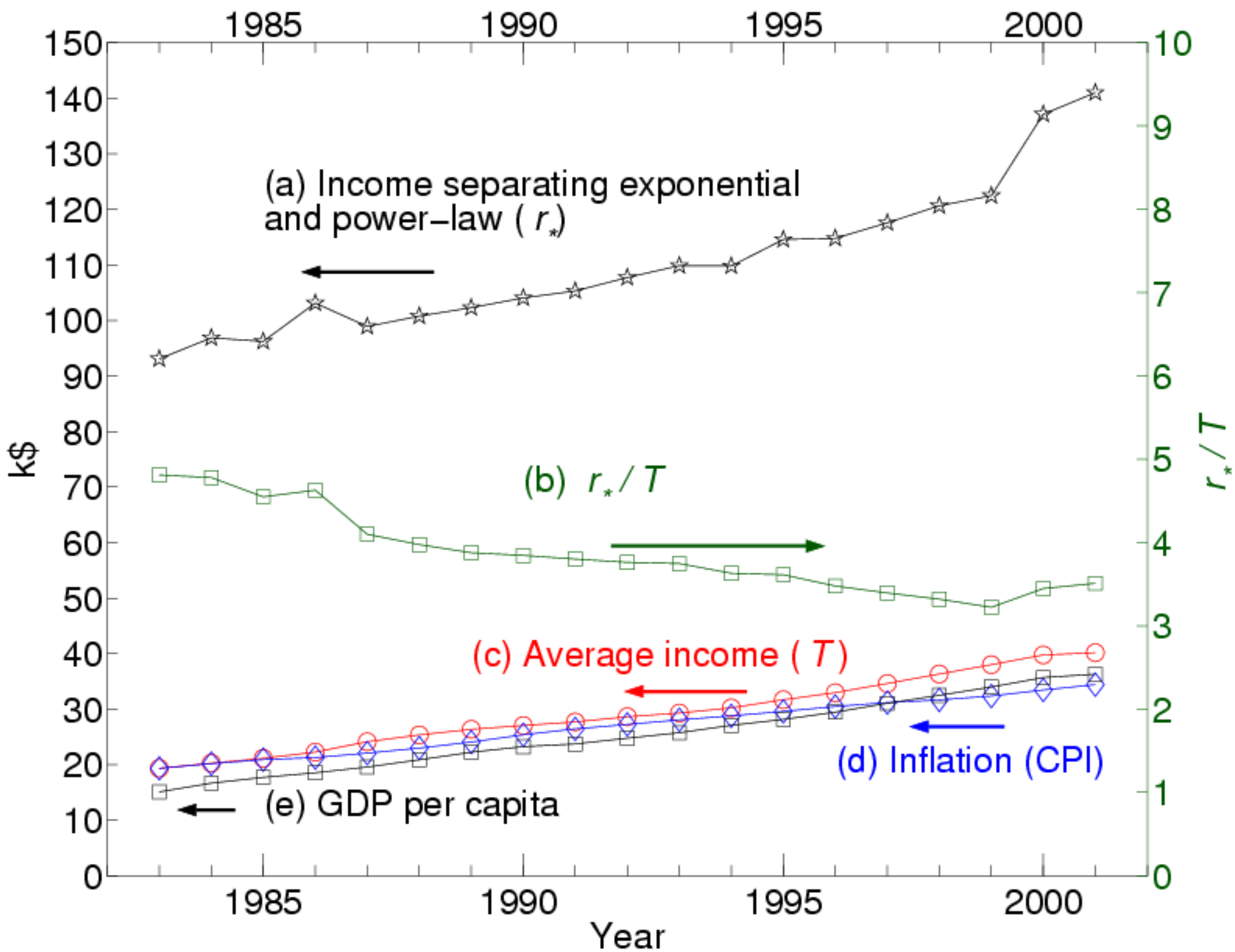


Lorenz curve is calculated for families $P_2(r) \propto r\exp(-r/T)$. The calculated Gini coefficient for families is G=3/8=37.5%

No significant changes in Gini and Lorenz for the last 50 years. The exponential ("thermal") Boltzmann-Gibbs distribution is very stable, since it maximizes entropy.

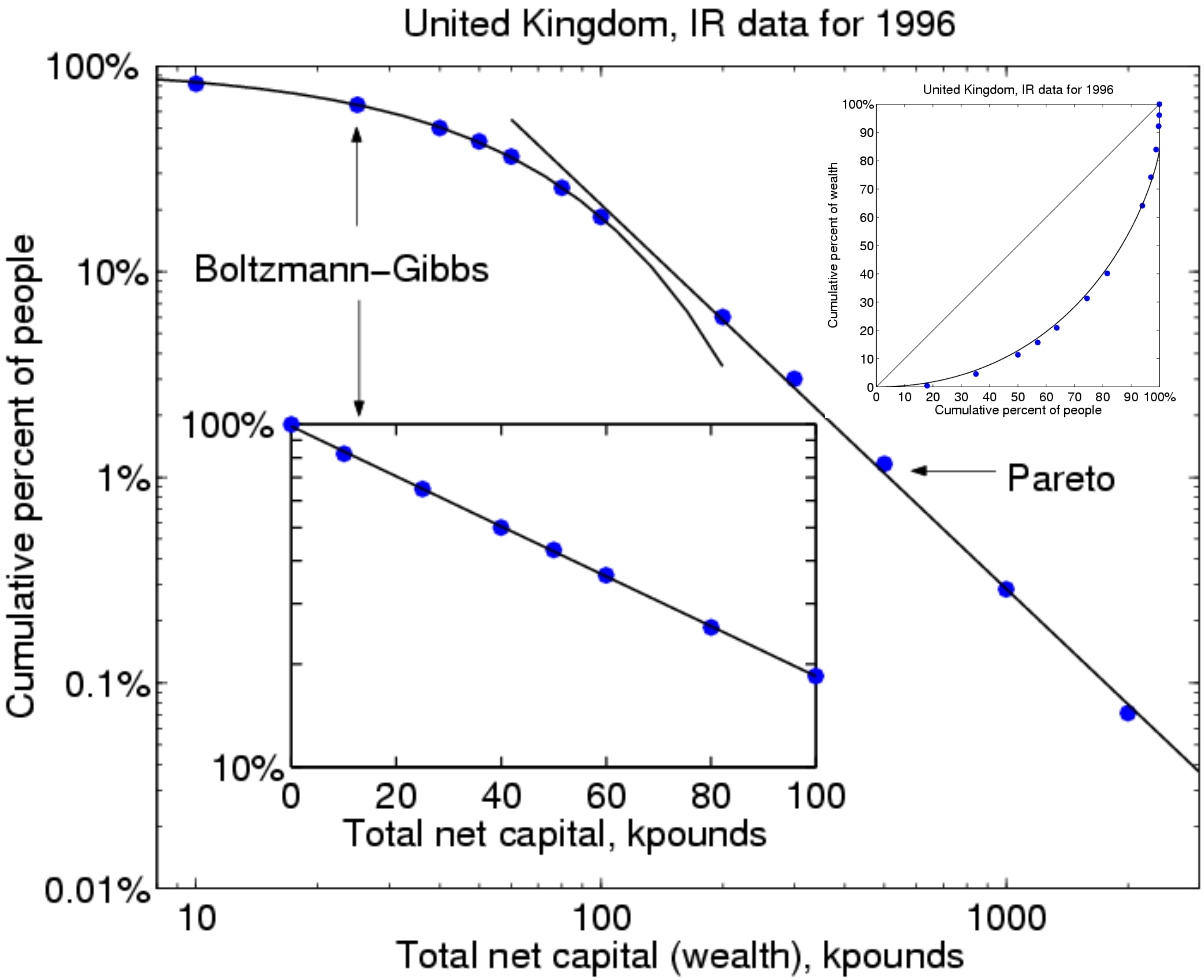
Maximum entropy (the 2nd law of thermodynamics) ⇒ **equilibrium inequality:**
G=1/2 for individuals,
G=3/8 for families.

Time evolution of income temperature



The nominal average income T doubled: 20 k$ 1983 40 k$ 2001, but it is mostly inflation.

Wealth distribution in the United Kingdom



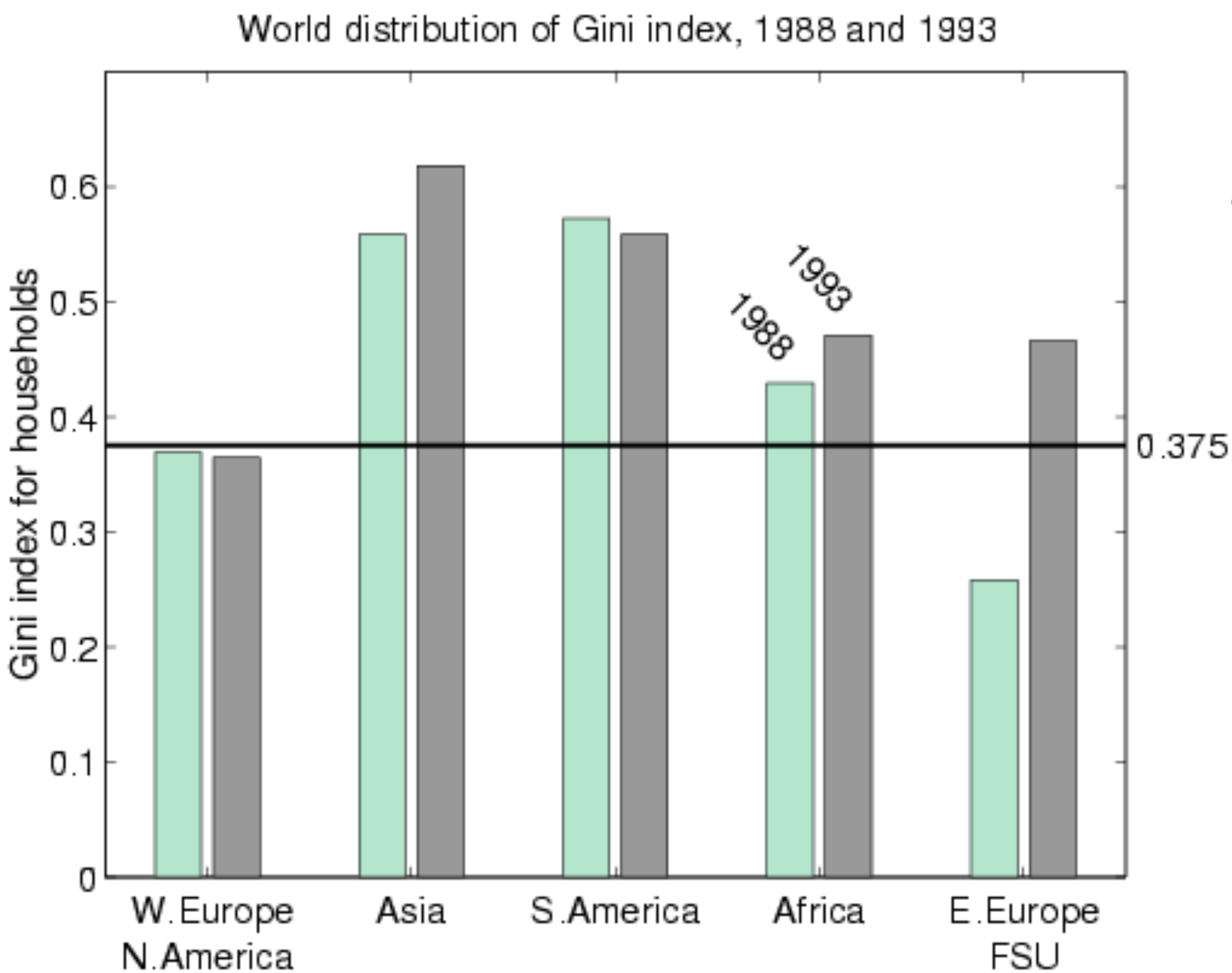
For UK
in 1996,
T = 60 k£

Pareto
index
α = 1.9

Fraction
of wealth
in the tail
f = 16%

World distribution of Gini coefficient

The data from the World Bank (B. Milanović)

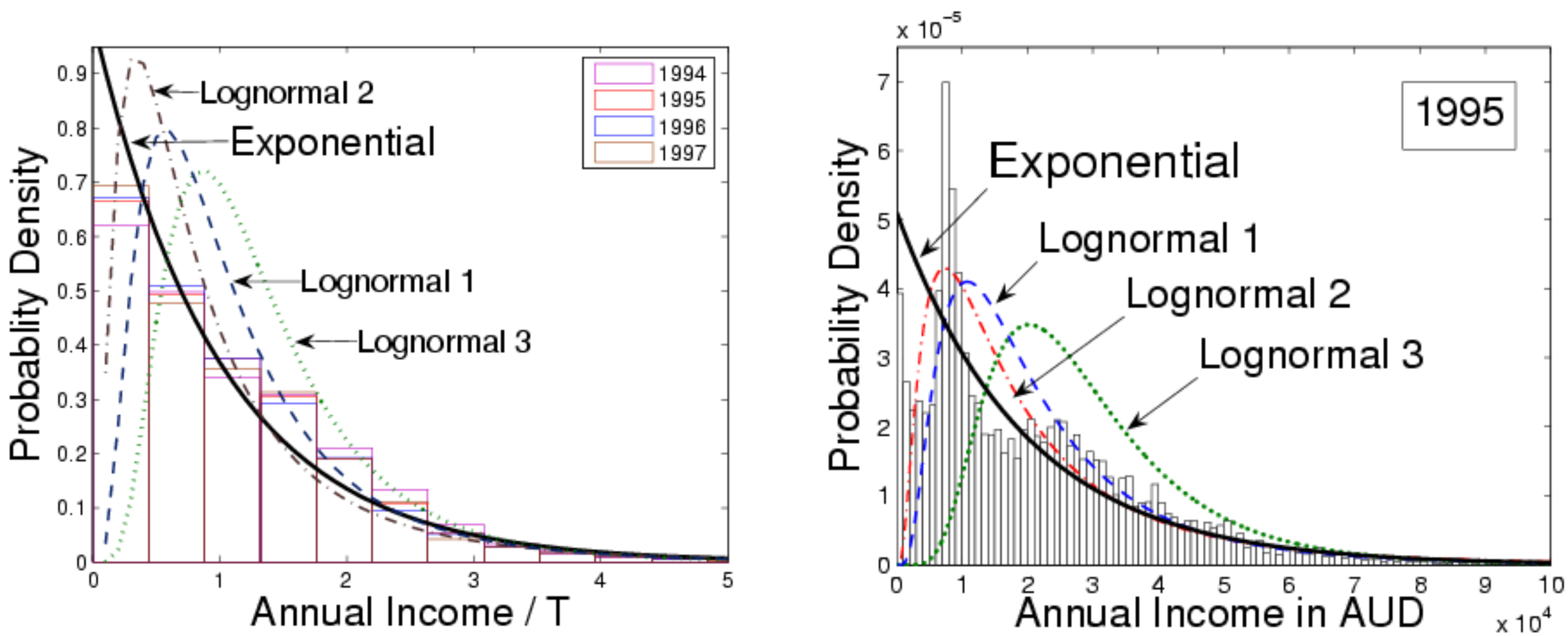


In W. Europe and N. America, G is close to 3/8=37.5%, in agreement with our theory.

Other regions have higher G, i.e. higher inequality.

A sharp increase of G is observed in E. Europe and former Soviet Union (FSU) after the collapse of communism – no equilibrium yet.

Income distribution in Australia



The coarse-grained PDF (probability density function) is consistent with a simple exponential fit.

The fine-resolution PDF shows a sharp peak around 7.3 kAU$, probably related to a welfare threshold set by the government.